Social and Emotional Learning in Mathematics

Guidelines for Curriculum Developers

Saroja Warner and
Heather Schwartz

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Introduction to the SEL in Mathematics Guidelines

The consensus from decades of research and practice is that attention to social and emotional learning (SEL) in academic areas leads to increased student achievement. Attending to SEL in the teaching and learning of mathematics specifically is increasingly recognized by both researchers and educators as a strategy for making sure that every student—especially those who have been historically underserved—has meaningful opportunities to learn and succeed.

Given the documented impact that SEL has on improving student achievement, curriculum developers can benefit from understanding how they can incorporate SEL into curricula to support improved engagement and better outcomes for all students. Accordingly, this document presents a set of guidelines designed to support curriculum developers in incorporating equity-centered SEL into high-quality mathematics curriculum materials.

CASEL and WestEd have served as partners to the Bill & Melinda Gates Foundation's K–12 Solutions team, identifying key characteristics of SEL in mathematics content and instruction and embedding them within these guidelines for high-quality mathematics instructional materials.

The guidelines are informed by several sources:

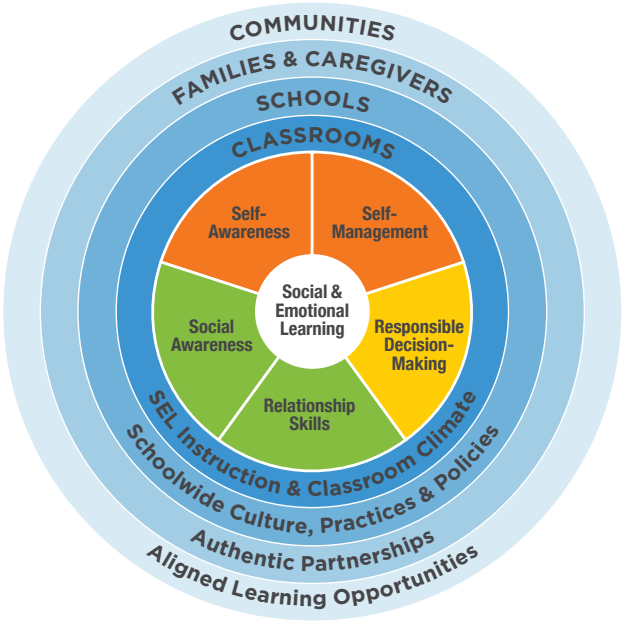
- the SEL frameworks established by CASEL and the WestEd-led Center to Improve Social and Emotional Learning and School Safety
- the research and evidence base for culturally responsive and sustaining education
- a landscape scan conducted in February 2023, which included an extensive review of other leading frameworks used in the field, local and state policies, research- and evidence-based practices, and high-quality instructional materials
- input and feedback from experts in the field (see Appendix B for details)

Overview of Social and Emotional Learning

SEL is the process through which young people and adults acquire and apply inter- and intrapersonal social and emotional competencies that support them personally and professionally. CASEL has elevated the importance of a systemic approach to SEL to align research, policy, and action (Mahoney et al., 2021). Systemic SEL implementation includes coordinating practices across classrooms, schools, families, and communities to enhance all students' social, emotional, and academic learning. CASEL's view of systemic SEL is visible within our framework for SEL (see Figure 1). At the center are the five core social and emotional competencies—broad, interrelated areas that support learning and development. Circling them are four key settings that are critical for student learning and development: classrooms, schools, families and caregivers, and communities.

A robust research base has established the central role of SEL in successful learning across content areas (Durlak et al., 2022; Mahoney et al., 2018). Social and emotional competencies such as relationship skills and self-awareness function as critical resources and tools for student engagement and achievement. Additionally, when integrated with academics, **SEL can help transform students' experiences of the classroom by putting them in the driver's seat as they set meaningful goals, grapple with content, and collaborate with peers—all within a supportive and challenging classroom environment** (Schwartz et al., 2023).

Figure 1. CASEL's Framework for Social and Emotional Learning



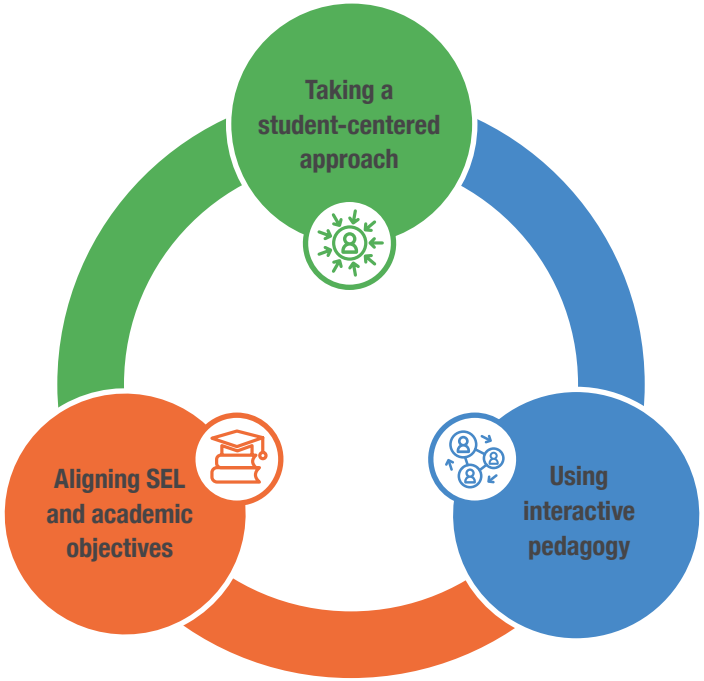
Source: CASEL, 2020

Within CASEL's systemic model (see Figure 2), the integration of SEL in academics falls into the following three main buckets:

- **Taking a student-centered approach** involves educators spending time getting to know their students. Educators thoughtfully plan instruction to support students with challenging, meaningful work no matter their current academic standing or disability status.
- **Aligning SEL and academic standards** assists with students' academic attainment by providing students with opportunities to learn about and practice the skills needed to meet classroom expectations. These skills include working independently toward goals and challenging the ideas of others in a way that is thoughtful and informed.
- **Using interactive pedagogy** to foster collaboration and reflection helps ensure that all students are speaking and listening every day as they engage in meaning making as valued members of the classroom community.

In 2019, Jagers, Rivas-Drake, and Williams shared a set of focal constructs aligned with CASEL's overarching core competencies. These focal constructs are **identity, agency, belonging, collaborative problem-solving, and curiosity** (see Figure 3). These constructs distill recent scholarship on creating learning environments that provide support and opportunity for all students (Jagers et al., 2019; Jagers et al., 2021). They are particularly relevant for guidance on supporting students' motivation, persistence, and engagement and have helped to shape the guidance in this document.

Figure 2. Three Interdependent Components of Integrating SEL Into Academics



Source: CASEL

Figure 3. Focal Constructs of SEL

Who we are and how we see ourselves impacts how we relate to academic material, whether we believe we can be successful, and how we navigate and perceive the world.

With **agency**, we understand that we can have an impact, and we choose to act to improve conditions for ourselves and others.

We all are best able to contribute when we feel we are valued, welcomed, and an **essential part of the community**.

Through **curiosity**, we learn more about ourselves, others, and the world and seek new information, which equips us to make responsible decisions.

We are more effective when we use relationship skills to authentically listen, better understand problems, and **work together to develop solutions**.

Source: CASEL

> "A lot of things happened during the pandemic that raised the need for [social–emotional learning]. … At the Dana Center we have gotten so many more requests for information about social–emotional learning, how to interweave that into academics, how to attend to that."
>
> —Curriculum Developer

The Need for Social and Emotional Learning Within Math Instruction

Over the past decade, there has been increased understanding of the central role that SEL plays in the growth and development of young people. Thousands of schools within the United States have implemented SEL programming, some using federal, state, and local funding granted for this purpose (Schwartz et al., 2022). Hundreds of studies have shown that in addition to supporting school connectedness for students, SEL leads to increased student achievement through engagement and higher levels of "school functioning" (Cipriano et al., 2023; Durlak et al., 2011; Durlak et al., 2022; Greenberg, 2023; Taylor et al., 2017). Leading voices and organizations in the field have described an increased need for SEL programming and for instructional materials and resources that integrate SEL within core content areas, including mathematics.[1]

The University of Texas at Austin's Charles A. Dana Center and CASEL (2016) describe SEL and mathematics as "inextricably linked." Unfortunately, commonly held beliefs about *natural* or *hereditary* ability (Moore & Shenk, 2017) and the perceived binary between *math people* and *non–math people* (Gaither, 2018; Palmer, 2009) minimize the critical role of identity and belonging in mathematics learning and achievement. **Students who feel a sense of belonging in their classroom learning communities are more likely to feel safe, take intellectual risks, share their thinking, ask questions, and be better positioned to learn.** When focal constructs of SEL are not addressed in mathematics instruction and curricular materials, students are likely to disengage and exhibit disproportionate academic and social struggles in their mathematics classrooms and instruction. Furthermore, many of the "habits of mind" that are core to math, such as generalizing, representing, and reasoning, are supported through SEL competencies.

> "When I think about high-quality instruction, I'm always thinking about a discussion, a way you can justify your point of view, the evidence that substantiates it. … But it always seems to happen in science, social studies, and [English language arts]. … In math, teachers are, like, 'But there's always one right answer.'"
>
> —School Leader

> "I love the idea of [relationship building]. … Even in places where we have a SEL curriculum, it's like, 'Oh these 10 minutes at the beginning of the day—[I've completed] my SEL, [and I'm ready to] move on to the next piece.' … [Guidance like this] really helps build those relationships, because relationships come before rigor."
>
> —Teacher

1 The direct quotes shared throughout this document were collected from focus groups that included some of these voices—practitioners, researchers, and leaders who provided feedback and insights on SEL in mathematics guidelines.

The SEL in Mathematics Guidelines

The SEL in Mathematics Guidelines address the following two principal needs for curriculum materials:

- **practical guidance that explicitly embeds SEL** approaches and skills into mathematics instruction and guides classroom practices
- **student-centered outcomes** and descriptions for curriculum developers and educators to look for when implementing guidelines-informed curricula

Key Learning and Insights From the Landscape Scan

As stated earlier, WestEd and CASEL conducted a landscape scan of frameworks, policies, practices, and materials related to SEL in mathematics. The goal of this scan was to identify key learning and insights to inform the SEL in Mathematics Guidelines. The following are a few of those key things learned.

- **The connection between SEL and academic attainment:** CASEL's five SEL competencies are the most frequently referenced and affirmed competencies in conversations about SEL and mathematics. States and districts frequently perform crosswalks in which they align mathematics practice standards with CASEL's five SEL competencies.
- **The role of adults:** Family engagement and early math exposure are predictors of long-term school success. Teachers' self-awareness and social awareness inform the implementation of culturally responsive mathematics instruction that helps every student reach their full potential.
- **The importance of mathematics identity:** Students' mathematics identity plays a fundamental role in shaping their self-concept and beliefs about what they can achieve as they navigate mathematics content and learning. The mathematics materials that CASEL and WestEd reviewed emphasize the importance of a growth mindset, which counters outdated notions of fixed mathematics ability. The interconnected focal constructs presented earlier in Figure 3 (i.e., identity, agency, belonging, collaborative problem-solving, and curiosity) can be harnessed to support student's growth in mathematics.

Using the Guidelines

The SEL in Mathematics Guidelines are intended to support curriculum developers as they build high-quality instructional materials and steward student achievement. The guidelines offer multiple entry points for curriculum developers. They should not be seen through an *all or nothing* lens because implementing some or most of the guidelines will significantly increase opportunities for students to achieve outcomes. While all the guidelines are grounded in research and offer meaningful ways to strengthen student learning, curriculum developers should thoughtfully prioritize which guidelines to follow based on their contexts and goals—ensuring the final product remains coherent, manageable, and impactful for both teachers and students.

When implemented strategically, a curriculum informed by these guidelines lays the groundwork for a mutually beneficial relationship between students' SEL and their academic success. *The guidelines include recommended instructional practices for teachers to implement that identify, affirm, and sustain students' assets.*

> "[These guidelines are] looking at what each student is bringing into the space of the classroom, and utilizing that to guide [instruction]. … [They] did a really good job of honing that."
>
> —School Leader

Three Central Tenets

The SEL in Mathematics Guidelines are organized by **three central tenets** that are aligned with essential characteristics of culturally responsive and sustaining math education. These three tenets indicate that SEL in mathematics curriculum materials must accomplish the following:

1. Affirm students' identities as learners
2. Promote agency and belonging
3. Build critical consciousness through collective responsibility

Each section of these guidelines defines and describes one of the tenets.

Student Outcomes

Student outcomes anchor the guidelines and illustrate how a curriculum informed by these guidelines can positively shape students' behaviors, actions, and skills.

The student outcomes describe short-term and/or long-term indicators of progress related to students' knowledge, skills, and attitudes. The guidelines are organized beneath the student outcomes they are most clearly designed to promote. Each guideline is followed by the aligned CASEL competencies. The competencies (introduced in Figure 1) and their corresponding focal constructs (introduced in Figure 3) are listed below:

- **Self-awareness** connects to **identity**.
- **Self-management** connects to **agency**.
- **Social awareness** connects to **belonging**.
- **Relationship skills** connects to **collaborative problem-solving**.
- **Responsible decision-making** connects to **curiosity**.

Contextualized definitions and meanings of key concepts found within this document are explained in Appendix A.

> "Offering opportunities for students to have those moments in a math classroom, to have those spaces to reflect, to be metacognitive, to consider their identity or their math story. … feels like the most powerful thing that could happen. Recognizing that teachers have learning to do. … The weaving of [SEL] is so critical, [as is] not having [SEL] be seen as a separate subject or something that happens outside of the math classroom."
>
> —Instructional Coach

Tenet 1: Learner Identity Development

Curriculum materials support all students in seeing themselves as capable, valued math learners. Lessons honor and value student, family, and community cultural and linguistic practices as vehicles for the acquisition and demonstration of knowledge, skill-building, and meaning-making. There are consistent opportunities for connection and relevance to students' lived experiences and future aspirations. Diverse representations of individuals, mathematical thinkers, and cultural perspectives—both global and local—are woven throughout to foster relevance, belonging, and engagement.

Personal Connections to Math (Student Outcomes 1 and 2)

1. **Students' math identities reflect an explicit awareness of their academic and cultural assets, their individual strengths and growth areas, and the way they work best with others.**
2. **Students demonstrate a deep understanding of everyday mathematics inside their homes, schools, and communities; within their cultural backgrounds; and across their interests.**

Tool: 10 Phrases to Build Positive Academic Mindsets. Use these phrases throughout the day to help build students' self-efficacy and affirm their place in the learning community.

Guidelines

» Use practices (e.g., getting-to-know-you surveys, take-home questionnaires) to **learn more about students' lived experiences and cultural assets** (e.g., traditions and values). (self-awareness, social awareness)

» **Value multilingualism** as a strength for understanding mathematical language (e.g., encouraging students to share linguistic translations in their sense-making, building on students' informal expressions of mathematical learning and understanding). (self-awareness, social awareness, relationship skills)

» **Invite students to make connections** between classroom learning and their valuable knowledge and experience from outside the classroom. (self-awareness, self-management, social awareness)

» **Apply math skills to real-world scenarios** that students are engaged in now, outside of school (e.g., cooking with parents or caregivers, building furniture, earning and/or saving money, understanding a graph in the news). (self-awareness, social awareness)

» Encourage students to **envision themselves as math learners and math *doers*** and to consider a range of **career pathways** that require mathematical skill sets (e.g., engineering, assembly line production, architecture, urban planning) or how their future aspirations intersect with mathematics. (self-awareness, self-management, social awareness)

» **Widen students' understanding of who is a mathematician** by highlighting culturally, racially, linguistically, and gender-diverse individuals (widely known or not) who have made significant mathematical contributions to society (e.g., Katherine Johnson, Benjamin Banneker, Adhara Pérez, Francis Su, Mary G. Ross), and support students to identify qualities and identities they have in common with mathematicians. (self-awareness, self-management, social awareness)

» Provide **explicit and specific feedback** about students' mathematical contributions, assets, and SEL competencies demonstrated (e.g., self-awareness, responsible decision-making skills) and individual areas for growth. (self-awareness, self-management, social awareness)

Growing Math Skills Over Time (Student Outcomes 3 and 4)

3. **Students understand the evolving nature of mathematics identities.**
4. **Students communicate their personal stories and experiences of math successes and growth throughout their lives.**

Tool: **Learner Autobiography**. This activity is an opportunity for students to explore their identities as learners and how past experiences have shaped their sense of themselves in the classroom. Reading these autobiographies will be an important learning experience for their teachers as well.

Guidelines

» Incorporate relevant stories and **storytelling within mathematics instructional delivery** and activities. (self-awareness, relationship skills, social awareness)

» Guide students to **reflect on how their feelings and experiences with math have changed over time**, from early experiences to instructional milestones, and address any past harm or misconceptions about their mathematical capabilities (e.g., self-affirmations, journal or discussion prompts). (**self-awareness, self-management, social awareness**)

» Provide opportunities for students and teachers to share about a time they overcame a challenge in a math class or other education setting and **emphasize the value of effort and growth**. Discourage notions of fixed ability (e.g., "I've always been good at math" or "I am not a math person"). (**self-awareness, self-management, social awareness**)

» Prompt teachers and students to **recognize and share their "math lineage"** (i.e., parents or family members who use mathematics in their daily lives, such as cooks, carpenters, electricians, and nurses). (**self-awareness, self-management, social awareness**)

» Provide guidance and materials (in multiple languages) for parents and caregivers to **support and extend learning in the home** and share their own stories of using math in life and improving skills through effort. (**self-awareness, social awareness**)

» Promote educators' self-awareness of their personal biases about students' capacity for math, and offer **strategies for challenging negative societal beliefs and stereotypes** that impact students' mathematics identity development. (**self-awareness, self-management, relationship skills**)

Lesson Plan Examples

Example 1: In this lesson, a Black female student is using math in her everyday life to achieve her leadership goals. Students use equations and inequalities to determine the most impactful combination of tabling and creating posters in her campaign for Class President. This connects to Tenet 1: Outcomes 1 and 2. (See Lesson C1 for full lesson plan.)

Example 2: In this lesson, students are asked to share numbers that represent important aspects of their identities, graph those numbers in groups, and reflect on how the numbers create social awareness. (See Lesson C2 for full lesson plan.)

Example 3: In this lesson, students are learning about ratio and proportion through exploring art and recipes from various cultures. As a bonus, they are invited to share and scale a family recipe and demonstrate their understanding of proportions. (See Lesson C3 for full lesson plan.)

Example 4: In this lesson, students are asked to think about experiences in their communities that involve a steady change or constant rate (e.g., rate of change in voter registration rates, growth of a popular artist's social media following). Then, the teacher introduces slope-intercept form using engaging and relevant examples. (See Lesson C4 for full lesson plan.)

Example 5: Teams of students use professional sports teams' stats, along with research on each player, to make predictions about future games. Students continue to work in teams to visualize the data and engage in comparative analysis. (See Lesson C5 for full lesson plan.)

Tenet 2: Agency and Belonging

When educators build classroom environments that prioritize agency and belonging, students understand they are valued members of the learning community and feel empowered to make choices and use their skills to impact things that are important to them. Guidelines that support educators in modeling and promoting curiosity, empathy, communication skills, and persistence in the math classroom set the tone for students to support each other's effort and success, build on one another's ideas, and see themselves as knowledge creators and math doers.

Strengthening Peer Community (Student Outcomes 1 and 2)

1. **Students demonstrate mathematical confidence and agency through their participation in classroom discussion and activities.**
2. **Students feel safe to engage in productive struggle, evidenced by their willingness to take risks, make errors, receive feedback, and revise their thinking. This includes students sharing their unfinished thinking and celebrating their progress and peers' work.**

Guidelines

- » Support **collaborative, peer-to-peer conversations** that invite curiosity and inquiry, build collective understanding, and offer a judgment-free space where students feel comfortable offering ideas and answers they are not sure about. (**relationship skills, self-awareness, responsible decision-making**)
- » Incorporate math activities and materials that provide **opportunities for play, exploration, and joy**. (**self-awareness, self-management, social awareness**)
- » Prompt teaching practices that leverage **group work and collaborative learning** that strengthen peer-to-peer communication and build a sense of belonging. (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)
- » Prompt student reflection and **discussion about the complex emotions involved in mathematics success**, such as frustration, vulnerability, and intellectual humility, and how emotions and mindsets affect one's learning experiences. (**self-awareness, self-management, social awareness, responsible decision-making, relationship skills**)

- Prompt students and educators to actively **recognize and reframe deficit-based thinking** and language about themselves and others. (**self-awareness, self-management, relationship skills**)
- Solicit **student feedback to educators** about learning needs and satisfaction with classroom relationships and experiences. (**self-awareness, self-management, social awareness, responsible decision-making**)
- Provide guidance for teachers to **partner with students to codevelop assessments and performance tasks** that demonstrate learning and progress toward lesson outcomes. (**self-management, responsible decision-making**)

Approaches to Problem-Solving (Student Outcomes 3 and 4)

3. **Students proactively use metacognitive strategies and can justify their mathematical understanding and describe their decision-making.**
4. **Students develop and exhibit their intrinsic motivation and emotions (e.g., joy, frustration, enthusiasm) toward mathematics learning, which is evident in their engagement levels, task endurance, and ability to sustain problem-solving stamina.**

Guidelines

- Encourage students to **use metacognition** to thoroughly describe their initial thinking, their problem-solving process, and the decisions and revisions they made to reach a solution. (**self-awareness, self-management, responsible decision-making**)
- Highlight the value of both arriving at the correct solution *and* engaging in the problem-solving process, and provide student **feedback and validation for their process** and their answer. (**self-awareness, responsible decision-making**)
- Describe ways teachers can **recognize and reinforce student ownership of mathematical thinking** and content (e.g., affirming student contributions and naming the mathematical elements in their responses, referencing student ideas and integrating student phrasing and contributions in their mathematical instruction and explanations). (**self-awareness, social awareness**)
- Cue teachers to validate student ideas and responses that are unanticipated or that differ from teachers' thinking and to **ask curious questions to clarify and understand students' rationale**. (**self-awareness, self-management, relationship skills, social awareness**)
- Incorporate **reflection, self-assessment, and review cycles** for students to solidify learning and make connections between past and new mathematical learning. (**self-awareness, self-management, relationship skills**)

Personal Reflection and Working Through Challenge (Student Outcomes 5, 6, 7, and 8)

5. **Students regularly engage in reflective practices that prompt them to take inventory of their capabilities, strengths, and areas of growth in relation to their mathematics achievement and goals.**
6. **Students demonstrate both self-directedness and flexibility in order to persist.**

7. **Students utilize strategies and/or seek assistance and resources that aid their self-management and produce healthy responses to math anxiety and overwhelm.**
8. **Students are explicitly aware of and can fully describe the SEL competencies embedded in their mathematics learning and can identify opportunities that target that particular skill or competency.**

Guidelines

- Incorporate **moments for self-awareness and reflection** in lessons and elevate these moments as an essential aspect of mathematics identity development rather than an optional activity. (**self-awareness, self-management**)
- Provide prompts for students to reflect on and **articulate their mathematical strengths and areas for growth**. (**self-awareness, self-management, social awareness**)
- **Support students to set meaningful and measurable goals** related to both their academic progress and their use of SEL competencies, and provide time for students to revisit goals and reflect on progress within weekly instruction. (**self-awareness, self-management, responsible decision-making**)
- Help students to **evaluate whether they need support** from a peer or teacher, know how to seek help, and decide when they need to take a break. (**self-awareness, self-management, responsible decision-making**)
- Support students to **take on a growth mindset** when they are grappling with a challenging problem and use **self-direction** to persist in their effort. (**self-awareness, self-management, relationship skills, responsible decision-making**)
- Offer strategies for self-management to effectively **address frustration and anxiety and to strengthen cognitive stamina** and persistence. (**self-management, responsible decision-making**)
- Prompt teachers to recognize when students are anxious and overwhelmed by math and respond supportively (e.g., offer a **break for movement or refocusing**). (**self-awareness, self-management**)
- Provide an overview of a unit or learning sequence, and **have students reflect on how they will need to apply SEL skills** throughout upcoming activities and assessments. (**self-awareness, self-management, responsible decision-making**)
- Prompt educators to **describe moments they used targeted SEL skills to solve a problem** or achieve success in math, and invite students to share their own stories. (**self-awareness, self-management**)

Lesson Plan Examples

Example 1: Before students explore linear and nonlinear functions, they have the opportunity to describe features of graphs using their own words. The teacher preplans language to support collaborative conversations. (See Lesson C6 for full lesson plan.)

Example 2: In this elementary lesson, the teacher introduces math vocabulary at the start of the lesson to support all learners. Additionally, students discuss why accountable talk is important before working in groups to solve long division problems. Then, each group creates a poster about how they solved one assigned problem for a class gallery walk. (See Lesson C7 for full lesson plan.)

Example 3: As part of a lesson on multiplication in which the teacher demonstrates different strategies, students have the opportunity to work independently and then explain and compare their mathematical approach with others to build their agency as "mathers." (See Lesson C8 for full lesson plan.)

Example 4: The teacher provides scaffolded opportunities for math talk about a scenario before students answer questions and graph data. Later, students use sentence stems to share their data and engage in the data presented by their peers. (See Lesson C9 for full lesson plan.)

Example 5: In this lesson, the educator builds student agency by reminding them that, as mathematicians, they each have a "toolbox" they can use. Students choose a tool to help complete a math problem and discuss as a class why they chose that tool. (See Lesson C10 for full lesson plan.)

Tenet 3: Collective Responsibility

Curricular materials provide opportunities for students to apply mathematical concepts to further their understanding of their relationships to others and to the world. **Civic readiness begins with building a classroom environment in which students understand themselves to be part of a community of learners engaged in collaborative problem-solving**. Then, in ways that are developmentally appropriate, students learn about the historical and current systems that impact global society and apply mathematical concepts and critical-thinking practices to real-world situations (local, national, global) in order to identify and/or apply solutions.

Collaborative Problem-Solving (Student Outcomes 1 and 2)

1. **Students possess and use interpersonal skills to engage in mathematical discourse and communicate their ideas and findings.**
2. **Students are partners in creating knowledge and experiences in the mathematics classroom and have opportunities to meaningfully contribute to, and build on, the ideas of others.**

Guidelines

» Develop regular routines and a scaffolded **structure for students to provide thoughtful, descriptive feedback and support** to one another about their mathematical thinking and work. (**self-awareness, self-management, social awareness, relationship skills**)

» Include tasks that give students opportunities to **formulate, communicate, and critique arguments** with peers and correct misunderstanding in a way that maintains positive relationships. (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)

» Prompt students to engage in **open-ended, mutual mathematical discourse** and to make connections among mathematical ideas and build on each other's thoughts (e.g., student to student, teacher to student). (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)

» Provide descriptive examples (e.g., written, audiovisual) of perspective-taking and guidance on **building counter-arguments** when engaging in mathematical discourse. (**self-awareness, self-management, social awareness, relationship skills**)

- Organize group activities that prioritize **working together to demonstrate progress and understanding** (e.g., through project-based learning and inquiry-based learning). (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)
- Include **real-world mathematical problems** that deliberately employ perspective taking in the problem-solving and collaborative activities as a means to build empathy and relationship skills. (**self-awareness, self-management, social awareness, relationship skills**)
- Use **technology** to expand students' mathematical learning, real-world application, and awareness of mathematical career and mentorship options. (**self-awareness, self-management, responsible decision-making**)

Mathematics Within Social Contexts (Student Outcomes 3 and 4)

3. **Students enact behaviors and practices that show they value and validate one another's lived experiences, cultural backgrounds, and identities.**
4. **Students utilize mathematical strategies, thinking, and language when discussing inequities and understand their application in examining and addressing societal challenges.**

Guidelines

- Foster the development of **positive understandings of individual and cultural differences** among students and how these assets contribute to the classroom whole. (**self-awareness, social awareness, relationship skills**)
- Facilitate critical **thinking and discussion about social identities** and how they can influence students' relationships to math. (**self-awareness, self-management, social awareness, relationship skills**)
- Foster the inclusion and **recognition of different ways of communicating**, including using multiple languages, nonverbal language, and movement to convey and share mathematical understandings. (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)
- Prioritize teacher practices and behaviors that **support participation by all students**, recognizing social cues and a spectrum of student comfort levels and honoring different forms of participation (beyond verbal). (**self-management, social awareness, relationship skills**)
- Encourage students to use their **lived experiences and current reality as context for their learning** and to be curious and empathetic about the experiences of their peers. (**self-awareness, self-management, social awareness, relationship skills**)
- Include supplemental artifacts such as videos, pictures, and case studies with **real-world data about local, national, and global challenges** to enhance learning activities through **simulation and authentic application**. (**self-awareness, self-management, social awareness, relationship skills, responsible decision-making**)
- Examine how **economic systems** influence society. (**self-awareness, social awareness, responsible decision-making**)

- Encourage and provide opportunities for **student-led math inquiries and research** into social phenomena and societal issues. (**self-management, social awareness, responsible decision-making**)
- Support teachers to develop **purpose statements** that reflect on their lived experience, reasons for becoming an educator, and deeply held beliefs about students and learning. (**self-awareness, self-management, responsible decision-making**)

Lesson Plan Examples

Example 1: In this elementary lesson, the teacher provides each student with a specific role to play during group work and ensures that students know how they can contribute. This helps each student understand themself as an important part of the learning process. (See Lesson C11 for full lesson plan.)

Example 2: In this middle school lesson, students use data to (a) better understand the needs of their peers and (b) make connections between collecting and analyzing data and making real positive change in their school community. This also provides opportunities to build on the thinking of peers and deliberate on open-ended questions. (See Lesson C12 for full lesson plan.)

Example 3: In this high school lesson on transforming graphs, the teacher provides students with multiple opportunities to clarify their explanations through conversation. This scaffolds their ability to support and challenge one another while practicing sense-making. (See Lesson C13 for full lesson plan.)

Example 4: Students use the Pythagorean Theorem to make decisions about the placement of a new supermarket after examining poverty rates at each of the three points and determining their distance from the Red Star Supermarket. (See Lesson C14 for full lesson plan.)

Example 5: In this high school lesson, students calculate values that are changing exponentially. Through conversations with peers, they explore real-world examples and discuss how they are similar and different mathematically. (See Lesson C15 for full lesson plan.)

Conditions for Success

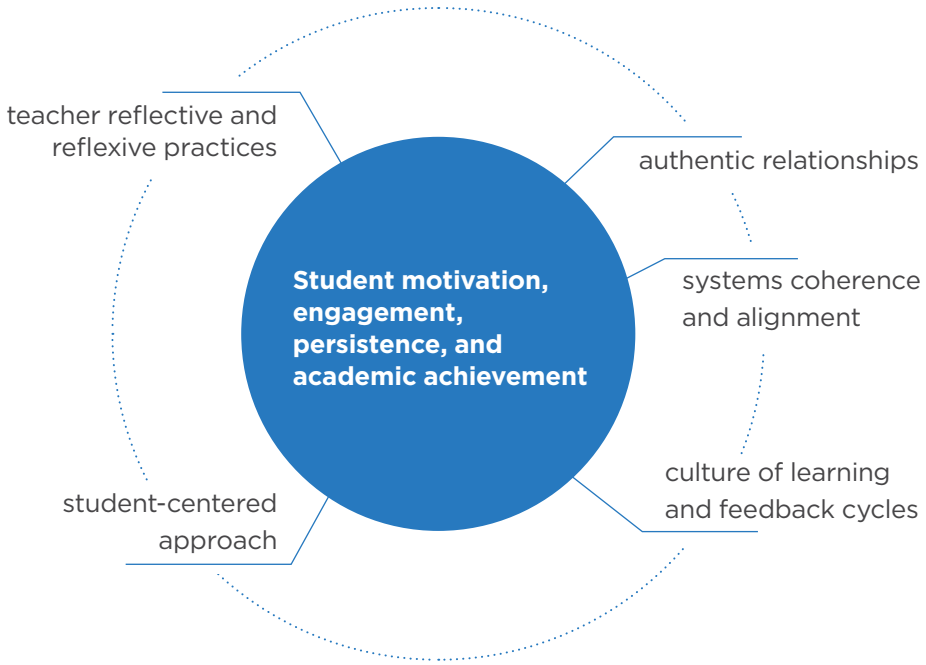
The curriculum is one crucial part of a larger equation for student success. Across focus groups (see Appendix B for details on the focus groups), participants made connections to other factors that play a fundamental role in students' motivation, engagement, persistence, and academic achievement:

- student-centered approach
- authentic relationships
- personal and learner identity development
- teacher reflective and reflexive practices
- culture of learning and feedback cycles
- systems coherence and alignment

These factors are aligned with **conditions for success** given their contextual relevance for the SEL in Mathematics Guidelines (see Figure 4). Both individually and collectively, the conditions operate as key levers that may strengthen students' motivation, engagement, persistence and academic achievement. Conditions such as these are conducive to the learning experiences and instructional rigor intended by the SEL in Mathematics Guidelines.

Please refer to Appendix A for contextual definitions and descriptions of many of the terms used in this section.

Figure 4. Conditions for Success



Student-Centered Approach and Authentic Relationships

Focus group participants enthusiastically described schools, classrooms, and educators that embodied the behaviors, skills, and knowledge necessary to implement a high-quality SEL in mathematics curriculum as being **student-centered** and prioritizing **authentic relationships** that include intentional partnerships with parents and caregivers as key contributors to students' social, emotional, and academic success.

Families whose students experience student-centered approaches and authentic relationships with educators and school personnel feel a sense of respect and belonging within the larger school community that is cultivated by school staff extending opportunities to contribute to student learning and classroom culture. There is a shared sense of trust and investment in equitable outcomes for students and the school as a whole.

Personal and Learner Identity Development

Learner identities, or the beliefs students hold about themselves as learners and their capabilities, significantly impact their motivation, engagement, and academic outcomes. To support positive learner identity development, students are regularly prompted to unpack their math stories, see themselves as part of a larger whole, and consider themselves a doer and learner of mathematics. Across all focus groups, there was consensus that attending to **identity development** was a necessity to unlock the capacity of student connectedness and belonging.

Teacher Reflective Practices

Educator self- and social awareness are essential to delivering high-quality mathematics instruction. To do so, routinizing **reflective practices** is essential. In the interest of building capabilities aligned with these guidelines for students and their families, educators and classroom teachers must proactively counteract practices and beliefs that conflict with those needed to advance success for all students despite geography and other factors.

Culture of Learning and Feedback Cycles

Establishing **a culture of learning and feedback** was repeatedly addressed and acknowledged as a baseline commitment to growth for schools and classrooms as they implement high-quality mathematics instruction. Community members, such as teachers and paraprofessionals, reflect this learner stance by seeing themselves on an individual and collective learning journey that requires professional learning and continuous improvement. In these spaces, adults and students operate from a growth mindset, value feedback, and engage in reflective practices about their pedagogy and academic performance.

> "Instructional materials that seem to presuppose that all of this material is for young people's sake and we are merely, as educators, the conduit of that process: I reemphasized that for social and emotional learning—that **we are not a conduit; we are participants in the moment**."
>
> —Field Expert

> "Teachers understand why this is important. … In my work, that's not the sell. … How will the structures of the school support the thinking work and the preparation? The training?"
>
> —Teacher

Systems Coherence and Alignment

Seeing oneself, whether staff or administrator, as part of a larger system is the first step in examining practices that impact student success. This underscores the need for **systemic coherence and alignment** across instruction and culture initiatives. Vertical planning among teacher teams and strategic timing for professional learning are strategies that build and reinforce this coherence. In a coherent and aligned delivery model, there is an overarching commitment to proactively work to mitigate the systemic inequities and barriers students, families, and those with less positional power face within the school community.

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Appendix A. Key Concepts in Practice

The excerpts in this section provide contextualized definitions of key concepts found within this document. Readers are invited to access the cited references to increase their accessibility and understanding of the SEL in Mathematics Guidelines.

Co-construction/Co-creation

"Educators can **co-create their classroom community with students** and define together how they agree to act and interact and how to be accountable to each other. This inclusive process goes against top-down, high-control models of classroom management educators often default to, especially in classrooms with Black, Latinx, Pacific Islander, Indigenous students and students from other historically marginalized communities. It also challenges adults to think more deeply about the values, biases, and implicit and explicit expectations they bring into the classroom—and which students those are serving, and which they are not, and why. By inviting students to collectively shape their own classroom community, educators can set the stage for real safety and belonging."

Center for Whole-Child Education. (2021, October 26). *How to co-create classroom culture with students*. https://turnaroundusa.org/how-to-co-create-classroom-culture-with-students/

Cultural Funds of Knowledge

"Classroom practice can be developed, transformed, and enriched by drawing upon the **existing funds of knowledge** in minority students' households. Funds of knowledge refers to those historically developed and accumulated strategies (e.g., skills, abilities, ideas, practices) or bodies of knowledge that are essential to a household's functioning and well-being."

González, N., Moll, L. C., Floyd-Tenery, M., Rivera, A., Rendón, P., Gonzales, R., & Amanti, C. (1994). *Funds of knowledge: Learning from language minority households*. Center for Applied Linguistics.

Culturally Responsive and Sustaining Education

"**Culturally responsive and sustaining education** is an approach to advancing learning and equity in education by creating culturally affirming and inclusive learning environments and experiences that support the attainment of comparably positive outcomes for all student groups."

Warner, S., & Browning, A. (2021). *What are social and emotional learning and culturally responsive and sustaining education—And what do they have to do with Critical Race Theory? A primer*. WestEd.

Justification

"Mathematical **justification** is the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense. Justification encompasses mathematical argumentation as expressed in the third of the Common Core State Standards for Mathematical Practice (SMP 3): Construct viable arguments and critique the reasoning of others (NGA Center and CCSSO, 2010)."

Bieda, K. N., & Staples, M. (2020). Justification as an equity practice. *Mathematics Teacher: Learning and Teaching PK–12*, *113*(2), 102–108.

Mathematical Authority

"The degree to which students are given opportunities to be involved in decision making and whether they have a say in establishing priorities in task completion, method, or pace of learning. Thus **[mathematical] authority** is not about 'who's in charge' in terms of classroom management but 'who's in charge' in terms of making mathematical contributions."

Gresalfi, M. S., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal*, *1*(1), 49–57. https://www.tandfonline.com/doi/abs/10.1207/s15544818ped0101_8

Metacognitive Strategies

"Teachers should pair instruction on cognitive strategies with that of **metacognitive strategies**—strategies that enable students to become aware of how they think when solving mathematics problems. This combined strategy instruction teaches students how to consider the appropriateness of the problem-solving approach, make sure that all procedural steps are implemented, and check for accuracy or to confirm that their answers make sense."

The IRIS Center. (n.d.). What evidence-based mathematics practices can teachers employ? Page 7: Metacognitive strategies [Course module]. *High-quality mathematics instruction: What teachers should know*. Peabody College, Vanderbilt University. https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p07/#content

Positionality

"**Positionality** means that important aspects of our identity, for example, our gender, our race, our class, our age … are markers of relational positions rather than essential qualities. Their effects and implications change according to context."

Tetreault, M. K. T. (1993). Classrooms for diversity: Rethinking curriculum and pedagogy. In J. Banks & C. A. McGee (Eds.), *Multicultural education: Issues and perspectives* (2nd ed., pp. 129–148). Allyn and Bacon.

Productive Struggle

"As students engage with a task, they must be mindful about the strategy they employ and assess whether it is productive. When they find they are at a dead end, they must be willing to abandon one strategy for another. When students labor and struggle but continue to try to make sense of a problem, they are engaging in **productive struggle**."

Pasquale, M. (2016). *Productive struggle in mathematics*. Interactive STEM Research + Practice Brief. Education Development Center. https://files.eric.ed.gov/fulltext/ED571660.pdf

Reflective Teaching

"**Reflective teaching** involves examining one's underlying beliefs about teaching and learning and one's alignment with actual classroom practice before, during and after a course is taught. When teaching reflectively, instructors think critically about their teaching and look for evidence of effective teaching."

Poorvu Center for Teaching and Learning. (2017). *Reflective teaching*. Yale University. https://poorvucenter.yale.edu/ReflectiveTeaching

Appendix B. Feedback From the Field

Feedback from the field was essential to ensuring that the SEL in Mathematics Guidelines reflect current research, understandings, and promising practices and match the identified needs. Two key strategies for feedback collection were internal reviews and focus groups.

Internal Review

Once the SEL in Mathematics Guidelines were drafted, they went through a content review that included WestEd subject matter experts Natalie Walrond, director of the Center to Improve Social and Emotional Learning and School Safety; Taunya Nesin, STEM Networking and Partnership director; and the project director, Dr. Saroja Warner, senior director of Culturally Responsive Systems. The revised document was shared in advance with focus group participants, coupled with curated pre-readings to prepare them for the discussion and feedback solicitation.

Focus Group Participants

Five role-alike groups—classroom educators, field experts and researchers, school leaders, instructional coaches, and curriculum developers—were convened for facilitated discussions and feedback on the guidelines. Focus group participants who have led adjacent, nationally recognized work were recruited from major organizations such as CASEL and University of Texas at Austin's Charles A. Dana Center and from school- and classroom-based practitioners from local districts across the country, with an emphasis on California, Florida, New York, and Texas. The following is a complete list of the focus group participants.

Marie Allred
Middle School Math Teacher
California

Brittany Atkins
Gifted and Advanced Learning Intervention Teacher
Maryland

Muna Beck
Primary STEM Achievement Specialist
Texas

Jennie Beltramini
Math Designer, Student Achievement Partners
New York

Kekai Bryant-Williams
Grades 6–12 Mathematics Teacher, Noli Indian School, Bureau of Indian Education
California

Frank "Trey" Csar III
Chief Operating Officer, Impact Florida
Florida

Dannielle Darbee
Principal, Brooklyn Academy of Global Finance
New York

Diana Fedderman
Director of Supplemental Educational Services
Florida

Hakim Johnson
Principal, Inspired Teaching Demonstration School
Washington, DC

Anna Katt
Instructional Coach
Washington, DC

Eugenio Longoria Sáenz
Director, Center for Family Math at NAFSCE
Texas

Tyrone Martinez-Black
Practice Integration Specialist, CASEL
Illinois

Nina Mauceri
Independent Instructional Consultant
New York

Susan May
Director of Curriculum, University of Texas at Austin, Charles A. Dana Center
Texas

Krystal Mieschberger
Assistant Principal of Instruction, Math, Engineering, and Science Academy (MESA) Charter High School
New York

Ben Mosley, EdD
K–8 School Principal
Maryland

Malea Mouton-Fuentes
Middle School Principal
California

LaShon Ormond
Chief Impact Officer, Amplify
Georgia

Chase Orton
Instructional Consultant
California

Deborah Peart
Math Education Consultant
Florida

Gregg Redmon
Special Education Teacher
New York

Claire Riddell
Middle Grade Educator, The Discovery School
Florida

Michelle Root
Professional Development Administrator on Special Assignment (AOSA)
California

Kwame Sarfo-Mensah
Founder and CEO, Identity Talk Consulting
Massachusetts

Alexandra Skoog-Hoffman
Director of Research–Practice Partnerships, CASEL
Illinois

Alexa Sorden
Founding Principal, Concourse Village Elementary School
New York

Kathy Sun
Assistant Professor and Director, Master's in Teaching and Teaching Credential Program, Santa Clara University
California

Yareli Vazquez
Math Instructional Specialist and Coach
Texas

Peter Walsh
Senior Program Manager, Agile Mind
California

Focus Group Commitments

The focus group work commitment comprised four pre-readings, a review of the guidelines document, and a 2-hour facilitated discussion with peers. The pre-readings provided a foundation regarding the framing and baseline understandings that informed the SEL in Mathematics Guidelines. They grounded participants' understanding of the core concepts, such as transformative SEL and SEL in mathematics, that informed the guidelines. Prior to the feedback session, participants were asked to conduct close readings of the guidelines document and note their feedback on individual document copies. Following the focus group sessions, individuals shared their edited copies with the WestEd team. All participants received an honorarium for their time and input.

Focus Group Pre-Readings

- *Social and Emotional Learning and Mathematics: Connections From the Field*, WestEd and Gates Learning Partnership Team (not publicly available)
- *Rooting Social and Emotional Well-Being Efforts in Equity: A Reflection Guide*, Natalie Walrond, Center to Improve Social and Emotional Learning and School Safety, WestEd
- *Transformative Social and Emotional Learning: In Pursuit of Educational Equity and Excellence*, Robert J. Jagers, Alexandra Skoog-Hoffman, Bloodine Barthelus, and Justina Schlund, CASEL
- *Integrating Social and Emotional Learning and the Common Core State Standards for Mathematics: Making the Case*, Charles A. Dana Center, University of Texas at Austin and CASEL

Focus Group Sessions

The WestEd team facilitated a 2-hour protocol with structured discussions that prompted rich conversations among role-alike peers. Participants were encouraged to share insights from their personal mathematics journeys, professional experiences, and understandings that are unique to their local contexts. Building on their prework, participants calibrated around major themes, things they noticed, and things they wondered about in the guidelines document. They were prompted to make recommendations to the content and structure of the document, identify blind spots, and suggest preferred language.

Focus Group Data

Following the last focus group session, the WestEd team synthesized findings from discussions and individual reviews of focus group transcripts and notes. Team members reviewed the recommendations and feedback provided by each focus group participant on their guidelines documents. The focus was to identify salient themes (Table B1) and outliers in feedback across role-alike groups. Through careful analysis of the data, the team identified relevant revisions and additions and noted areas to clarify for the revised guidelines document. Additionally, recommendations were gleaned for a future iteration of the document that falls outside the current scope, as these changes would warrant additional rounds of focus groups and reviews.

Table B1. Salient Themes From Role-Alike Focus Groups

Role-alike group	Key themes and recommendations
Classroom educators	• High-quality math instruction is grounded in an asset perspective, uplifting student personal, family, community, and cultural experiences. • Strong relationships are a precursor to student engagement and achievement. Relationship-building opportunities should be prioritized in the content. • There is an opportunity to more clearly emphasize teacher humility and the belief that students are co-creators who have mathematical authority (i.e., teachers don't own the answers). • The term "identity-affirming" may connote that students already have formed a sense of self. Cultural, social, and academic identities are fluid and evolving and should be framed as such. Mathematics instruction has the potential to support and affirm students as they explore and form their identities, including their math identity.
Field experts and researchers	• These guidelines are timely, as the postpandemic attentiveness to student and community well-being has driven more requests for materials and resources about weaving SEL into academics. • The field is rife with toolkits, resources, and information to support equitable, culturally responsive teacher instruction. These guidelines are a useful framework that leaves room for educator and school leader agency and decision-making. • Consider making connections to existing frameworks such as Complex Instruction and The Math Pact series (available from Corwin Press, a SAGE Company) that embody equitable, social and emotional mathematics instruction. • Guidelines should reinforce students' mathematical authority, teacher empathy, and opportunities for cross-curricular collaboration among all educators.

Role-alike group	Key themes and recommendations
School leaders	• There is a shortage of high-quality instructional materials that strike a balance between procedural and conceptual mathematics. Students should be able to "speak math," explain procedures or formulas, and understand how concepts are applicable to the real world. • Deeper thinking through discourse, justification, and conceptual understanding seems to show up more frequently in science, social studies, and ELA classrooms, whereas math teachers are more likely to point students toward the "correct answer." • Leaders expressed concerns about the resources necessary to support the teachers' capacity to implement guidelines-informed curricula. Within this recommendation was the request to be mindful of teacher experience levels. • The Collective Responsibility guidelines felt the most tangible, concrete, and clear for most school leaders.
Instructional coaches	• Instructional coaches function on a systematic level—coordinating and allocating right-sized supports that address varying teacher needs—and reach for curriculum supports that provide the agility and flexibility their work requires. • The math classroom should look different than it has in the past (e.g., flipped classrooms, collaborative work, stations, small-group pullouts, multiple modalities). • High-quality math instruction must be differentiated and personalized to include a "culture of error and growth," discourse, iteration, and student advocacy. • Participants expressed concern about fidelity of implementation and continuity given the multiple challenges teachers face (e.g., teacher shortages, capacity levels, testing, and accountability).
Curriculum developers	• Curriculum materials should be embedded with educative features that allow teachers to effectively engage in self-reflection and identity affirmation, considering the level of math trauma and negative experiences many teachers and students have. • Measures of effectiveness should be a part of the curricular package so that teachers can demonstrate successful implementation of identity affirmation, agency and belonging, and collective responsibility. • These student-centered guidelines can powerfully influence and support student exploration, metacognition, and reflection within the math classroom regardless of teacher mindsets, beliefs, and readiness. • Closer alignment with the Standards for Mathematical Practice (SMPs) can support districts, schools, and classrooms that have started to implement SEL in mathematics and need more support beyond crosswalking.

Overwhelmingly, the guidelines were well received and affirmed across focus group participants. In the closing survey, 100 percent of participants agreed that the guidelines were relevant to high-quality mathematics instruction. Almost all participants (96%) agreed that guidelines-informed curricula address a much-identified need in the field regarding the integration of SEL in mathematics and the role of identity, agency, and belonging in mathematics learning. They shared an appreciation for the student outcomes, which helped them visualize student benchmarks.

Similarly, there was consensus regarding concerns about guidelines-informed curricula. The emphasis on teacher capabilities to enact practices as presented in the guidelines is predicated on a readiness and preparation to engage in a mathematics curriculum that embraces an expanded understanding of mathematics instruction, one that includes identity work, agency, and belonging. Teacher pedagogical approaches surfaced as a critical element in achieving the student outcomes, and the quandary regarding the role of curricula was perplexing for most of our focus group participants. Identifying a throughline between curricula and pedagogical methods is key to realizing the student outcomes. Within this feedback came requests to include student examples, teacher reflection prompts, and aligned practices within the guidelines.

Participants applauded the student-centered and asset-based approaches within the guidelines, especially content that supports the development of a positive self-concept and recognizes the value of one's cultural and academic assets. Across focus groups, there was consensus that experiencing this type of curriculum stewards a belongingness and growth mindset for students. Related to students, there was a shared sentiment that making mathematical authority more explicit within the guidelines would remind teachers that learning spaces are co-constructed with students. For example, participants noted that when teachers ask students to identify their problem-solving strategies, students practice their mathematical decision-making and utilize their agency within their learning.

Appendix C. Sample Lesson Plans

Lesson C1. Tenet 1: Algebra 1—Reasoning With Equations and Inequalities

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns to Tenet 1, which is described in Table 1.

Lesson C1: Table 1. Overview of Tenet

Essential tenet	# 1: Identity Affirming
Student outcomes	2. Students learn and are able to demonstrate a deep understanding of everyday mathematics inside their homes, schools, and communities; within their cultural backgrounds; and across their interests.
Guideline	e. Guide teachers to incorporate opportunities for students to understand how math is relevant in their daily lives and throughout their communities (e.g., identifying how math is used in parents' or caregivers' careers/businesses or using maps and artifacts that include familiar points of reference, like students' neighborhoods or home countries). (self-awareness, self-management, social awareness)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C1: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	CCSS
Domain	Reasoning With Equations & Inequalities
Content standards	**A - REI.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A - REI.11:** Explain why the x-coordinates of the points where the graphs of the equations y = *f*(x) and y = *g*(x) intersect are the solutions of the equation *f*(x) = *g*(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). **A - REI.12:** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Component	Details
Source	CCSS
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 4. Model with mathematics. SMP 6. Attend to precision.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C1: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The ability to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Self-Management:** The ability to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
Example(s)	• When confronted with a potentially challenging word problem, students are able to manage their emotions and work through frustration and confusion without getting disengaged or having their feelings lead to less focused thinking.

Lesson Overview and Goals

We make sense of how to simultaneously consider multiple constraints to determine what is/is not feasible and also how to select from a set of feasible options to choose one (or more) that will help us achieve (or avoid) a particular outcome.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C1: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to graph the feasible region for a set of inequalities, identify intersectional points on the boundary of the feasible region, and evaluate the points to determine which point(s) maximizes the value given a set of real-world conditions.
Key vocabulary	Feasible region, inequality, intersection
Prior knowledge	1. Writing inequalities from a word problem 2. Graphing inequalities 3. Finding the feasible region on a graph of inequalities 4. Identifying the intersection of two lines 5. Evaluating algebraic expressions

Lesson Procedures

Directions/Script
Launch (10 minutes)
Have students use a think-pair-share to explore the concept of "feasibility." Listen to conversations and consider discussing with the full class where there are differences of opinion about what is/is not feasible. Teacher can ask students to generate other ideas about situations or scenarios that are/are not feasible. Teacher might also ask students to identify two different things that are feasible when done separately but not together (e.g., baking and riding a bike) and two things that are feasible to do together (e.g., watch TV and send a text). This begins to introduce the idea of a feasible region.

Directions/Script
Have students think-pair-share (possibly with a different partner) to see how many ways they can describe the feasible region, which might include the following: • The area where all of the colors overlap • The darkest area on the graph • The triangular area on the graph • The part of the graph that is on/above the red line, on/to the left of the orange line, and on/below the purple line.
Have students find points of intersection between the three different inequalities. Teacher might demonstrate how to find one point of intersection by setting two inequalities equal to each other, solving for x, and then substituting the value of x back into an equation to solve for y.
These two problems provide students with an opportunity to "test" points and determine whether they are/are not inside the feasible region. Try soliciting different opinions about whether your students agree or disagree with Josie/Marvin and have them explain/defend their reasoning. Teacher should ask students to identify at least one point that is within the feasible region.
Explore (30 minutes)
Students should be familiar with graphing inequalities and finding feasible regions. But they may need support with writing inequalities. Some prompts to help with this step include the following: • How many variables are there to consider? What are they? What letters should we use to represent them? • What are the limits that you are constrained by in this scenario? • How many constraints must be followed, and what are they (e.g., there are two "at most" and one "at least" constraints)? If students are stuck after writing and graphing the inequalities, additional prompts might include the following: • What is an example of a combination of Posters and Tabling that is feasible? How many votes would that combination get? • Repeat this step with a different combination. • What are some combinations that have the largest values for T and/or P (hint: they are at the intersections on the boundary of the feasible region)? • How can we find the points (combinations) where the inequalities intersect?
Debrief and closure (10 minutes)
The prompts on this slide are intended to push students' mathematical/critical thinking and might be shared/discussed with some students who complete the activity more quickly. Rather than having clear answers, these prompts are intended to promote discussion in which students have to justify their answers and might arrive at very different (correct and incorrect) ideas.

Directions/Script
The purpose of this slide is to explore what "inequalities" look like in real life. The graph shows how people serving life in prison without parole (by race) are disproportionate to their representation in society. Some additional prompts for the discussion might include the following: • What would a more equitable version of this graph look like? • What other inequalities might people experience that could contribute to this graph? » Race-based disparities in policing » Economic inequality » Inequitable access to legal counsel » Racially biased juries • A reasonable issue for students to raise (or that the teacher could raise) is what relevant information is missing from this graph. For example, it doesn't capture murder rates (the most common reason for getting life without parole) by race. However, this question can be further teased out by discussing some of the inequalities that could lead to disproportionate rates of violent crime committed. The reality is that Black men are 7 times more likely than White men to be wrongly convicted of murder and receive 19 percent longer sentences than White men from similar backgrounds for the same crimes.

Lesson C2. Tenet 1: Middle Grades—Statistics and Probability

This lesson was created by a teacher during a design lab focused on social and emotional learning (SEL) and culturally responsive and sustaining education (CRSE). The lesson aligns with Tenet 1, which is described in Table 1.

Lesson C2: Table 1. Overview of Tenet

Component	Details
Essential tenet	#1: Identity Affirming
Student outcomes	2. Students learn and are able to demonstrate a deep understanding of everyday mathematics inside their homes, schools, and communities; within their cultural backgrounds; and across their interests.
Guideline	1.a Prompt educators to prioritize authentic relationship building and enact practices (e.g., getting-to-know-you surveys, take-home questionnaires) to learn more about students' lived experiences and cultural assets and embed that understanding within instruction and classroom culture in ways that are relevant to students' home life (e.g., routines, traditions, and values) without trivializing or stereotyping students' backgrounds. (self-awareness, social awareness)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C2: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Source: Common Core State Standards
Domain	
Content standards	**7.SP.C.8b:** Students explore student identities and cultural backgrounds while fostering connection and self-expression with number of languages spoken, number of countries visited, number of sports played, family traditions celebrated, and favorite books read using real data from students in the classroom.
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 2. Reason abstractly and quantitatively. SMP 3. Construct viable arguments and critique reasoning of others. SMP 8. Look for and express regularity in repeated reasoning.

Tables 3 and 4 list the SEL competencies to which this lesson aligns for both students and teachers.

Lesson C2: Table 3. SEL Competencies for Students

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The ability to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Relationship Skills:** The ability to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
Example(s)	• Integrating personal and social identities • Identifying personal, cultural, and linguistic assets

Lesson C2: Table 4. SEL Competencies for Teachers

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The ability to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Responsible Decision-Making:** The ability to make caring and constructive choices about personal behavior and social interactions across diverse situations.
Example(s)	• (Social) awareness of student struggles and challenges • Actively listening to student responses and demonstrating curiosity and open-mindedness

Lesson Overview and Goals

We reason that fair distributions, the concept of ratio, and fractions are about belonging to sets and wholes.

Lesson Setup

Table 5 provides some key information to help teachers set up and carry out the lesson.

Lesson C2: Table 5. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/ resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to determine means, median, mode, and range of data based on students' responses and experiences to collect data.
Key vocabulary	Means, median, mode, and range
Prior knowledge	The YouTube video Means, Median, Mode, and Range can be used to clarify misconceptions students have about means, median, mode, and range with a set of data.

Lesson Procedures

Directions/Script
Launch (10 minutes)
Here's a 45-minute middle school mathematics activity that incorporates SEL with cultural relevance, focusing on student identities, culture, and lived experiences. **Activity: "Our Stories in Numbers"** **Objective:** Use mathematics to explore student identities and cultural backgrounds while fostering connection and self-expression. **Materials Needed:** • Chart paper or whiteboards • Markers • Graph paper • Calculators (if needed) • Sticky notes **Activity Outline** **1. Introduction (5 minutes)** • **Discussion Prompt:** Begin with a brief conversation about how numbers and statistics can represent our lives and experiences. Ask students: » "What numbers tell your story?" (e.g., number of family members, languages spoken, years lived in a community) • **Cultural Connection:** Explain that this activity will help us see the math behind our identities.

Directions/Script
Explore (35 minutes)

2. Personal Data Collection (10 minutes)

- **Instructions:** Have each student take a few minutes to think about and jot down five numbers that represent important aspects of their identity or culture. Examples might include
  - » Number of languages spoken
  - » Number of countries visited
  - » Number of sports played
  - » Family traditions celebrated
  - » Favorite books read
- Sharing: If comfortable, students can share one or two of their numbers with a partner or small group.

3. Graphing Our Stories (15 minutes)

- **Group Work:** In small groups, students will choose two or three common numbers from their personal data. They will then do the following:
  - » Create a bar graph or pie chart that visually represents these numbers.
  - » Discuss why they chose these particular numbers and what they signify about their identities or backgrounds.
  - » Find means, median, mode, and range to expand on discussions.
- **Cultural Relevance:** Encourage students to think about how their numbers reflect their cultural experiences or family traditions.

4. Reflection and Discussion (10 minutes)

- **Gallery Walk:** After completing their graphs, groups can display their work around the classroom. Students will walk around to view each other's graphs.
- **Discussion Questions:**
  - » What patterns do you see in our class?
  - » How do these numbers help us understand each other better?
  - » In what ways do our backgrounds influence how we approach math and learning?

Directions/Script
Debrief and Closure (10 minutes)
5. Closing Reflection (5 minutes) • **Exit Ticket:** Ask students to write one takeaway from the activity on a sticky note and place it on a designated board. • **Final Thought:** Emphasize that our diverse backgrounds enrich our learning environment and that math is not just numbers but also stories and experiences. **Optional Extensions (if time allows)** • **Personal Math Story:** As a follow-up activity, students could write a short narrative or create a presentation about how their personal numbers relate to their experiences and culture. • **Connecting to Math:** Explore how statistics about the class (e.g., averages, ratios) can relate to students' identities, reinforcing the connection between math and real life.

Elevating SEL in the Lesson

In the "explore" section of this lesson, students work collaboratively and draw on the following SEL competencies:

- Self-awareness for understanding and articulating their thinking and for identification of feelings, strengths, and challenges
- Responsible decision-making for perseverance in problem-solving
- Relationship skills necessary for positive and harmonious engagement with peers in group discussions

Lesson C3. Tenet 1: Middle Grades—Proportions

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 1, which is described in Table 1.

Lesson C3: Table 1. Overview of Tenet

Essential tenet	#1: Promote opportunities for connection and relevance to students' lived experiences
Student outcomes	Students demonstrate a deep understanding of everyday mathematics inside their homes, schools, and communities; within their cultural backgrounds; and across their interests.
Guideline	**Invite students to make connections** between classroom learning and their valuable knowledge and experience from outside the classroom (self-awareness, self-management, social awareness).

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C3: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Source: Florida's BEST Standards for Mathematics
Domain	Proportions
Content standard	**MA.7.AR.3.2:** Apply previous understanding of ratios to solve real-world problems involving proportions.
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 2. Reason abstractly and quantitatively. SMP 4. Model with mathematics. SMP 6. Model with mathematics.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C3: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Relationship Skills:** Collaborating effectively in groups. • **Social Awareness:** Recognizing and appreciating diverse cultural perspectives.
Example(s)	• Working with peers to learn about different family recipes.

Lesson Overview and Goals

» Students will understand and apply the concept of proportions in real-life contexts.

» Students will explore cultural representations of proportions (e.g., cooking, art, community planning).

» Students will collaborate in groups, enhancing relationships and social skills.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C3: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Student outcomes	• Students will understand and apply the concept of proportions in real-life contexts. • Students will explore cultural representations of proportions (e.g., cooking, art, community planning). • Students will collaborate in groups, enhancing relationships and social skills.
Key vocabulary	Ratio, proportional, equivalent
Prior knowledge	Students understood ratio and unit rate concepts and used them to solve problems.

Lesson Procedures

Directions/Script
Launch (10 minutes)
1. **Warm-up Discussion:** • Ask students, "What does it mean to be proportional?" • Engage them in a brief discussion about how they see proportions in everyday life (e.g., cooking, art, sports). 2. **Cultural Connections:** • Introduce the idea that different cultures use proportions in unique ways (e.g., recipes, architecture). • Briefly share examples from various cultures, inviting students to contribute their own experiences.
Explore (50 minutes)
1. **Group Activity:** • Divide students into small groups and provide each with recipe cards from different cultures. • Ask them to identify the proportions in the recipes and discuss how these might change based on the number of servings (e.g., scaling up or down). • Each group will also explore how these recipes reflect cultural practices. 2. **Cultural Art Proportions:** • Show visual examples of artwork (e.g., Islamic geometric patterns, African textiles) that use proportions. • Ask groups to discuss and identify the proportional elements within the artwork.
3. Proportion Problems: • Hand out worksheets with problems related to the cultural contexts discussed. For example: » "If a recipe serves 4 people using a ratio of 2 cups of rice to 1 cup of water, how much rice and water do you need for 10 people?" » "In a painting that uses a 2:1 ratio of height to width, what would be the dimensions of the painting if the width is 4 feet?" **4. Present Findings:** • Each group presents one recipe and one art example, explaining the proportions used and their significance in the cultural context. • Encourage students to reflect on what they learned about relationships and collaboration in their groups.
7th Grade Math: Proportions in Cultural Contexts Worksheet (Located on the next page)
Debrief and closure (10 minutes)
• Ask students to write a short paragraph on how working with their group helped them understand the concept of proportions better. • Discuss how understanding different cultures can enhance our appreciation of math and relationships in our communities.

7th Grade Math: Proportions in Cultural Contexts Worksheet

Name:
Date:

Part 1: Recipe Proportions

You will be using the recipe cards provided by your teacher. Use the proportions in the recipe to solve the following problems:

1. Scaling a Recipe

The original recipe serves 4 people and uses the following ratio of ingredients:

» **2 cups of rice** to **1 cup of water**

Problem A:
If you need to serve 10 people, how many cups of rice and water will you need?

Problem B:
What if you only need to serve 2 people? Adjust the proportions accordingly.

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2. Creating Your Own Recipe

Imagine you have a recipe that serves 6 people. The recipe uses the following ratio of ingredients:

» **3 tablespoons of oil** to **2 teaspoons of salt**

Problem A:
How many tablespoons of oil and teaspoons of salt will you need if you're serving 12 people?

Problem B:
What if you're making this recipe for only 3 people? How much oil and salt will you need?

Part 2: Proportions in Cultural Art

Look at the examples of art that your teacher has provided. These artworks use proportions to create patterns and shapes. Answer the following questions:

1. Proportional Patterns in Art

In a geometric pattern, the ratio of large triangles to small triangles is 3:2.

Problem A:
If there are 15 large triangles, how many small triangles are there?

Problem B:
If the pattern has 24 small triangles, how many large triangles should there be?

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2. Art Dimensions

In a painting, the height-to-width ratio is 2:1.

Problem A:
If the width of the painting is 6 feet, what is the height of the painting?

Problem B:
If the painting's height is 8 feet, what is the width?

Part 3: Reflection

1. **Group Reflection:**

In a few sentences, describe how working with your group helped you better understand proportions. How did you collaborate to solve the problems?

2. **Cultural Connections:**

How did the cultural examples (recipes, art) help you understand the importance of proportions in everyday life? Write a short paragraph explaining your thoughts.

Bonus:
Do you have a favorite family recipe? Write down the ingredients and their proportions. Then, try to scale the recipe to serve a larger or smaller group of people!

Lesson C4. Tenet 1: Algebra 1—Reasoning and Linear Functions

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 1, which is described in Table 1.

Lesson C4: Table 1. Overview of Tenet

Essential tenet	# 1: Identity Affirming
Student outcomes	2. Students learn and are able to demonstrate a deep understanding of everyday mathematics inside their homes, school, and communities; within their cultural background and across their interests.
Guideline	e. Guide teachers to incorporate opportunities for students to understand how math is relevant in their daily lives and throughout their communities.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C4: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	TEKS: Algebra 1
Domain	Algebraic Reasoning and Linear Functions
Content standards	**A.3(C):** Determine the rate of change of a linear function represented tabularly, graphically, or symbolically in context. **A.3(D):** Graph linear functions on the coordinate plane and identify key attributes, including xxx- and yyy-intercepts, zeros, and slope, in mathematical and real-world problems. **A.3(E):** Write equations of linear functions given a table of values, a graph, and a verbal description. **A.2(B):** Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete. **A.2(I):** Write and solve equations involving direct variation, interpreting the constant of proportionality as the slope of the related linear function. **A.12(E):** Interpret and make predictions based on the mathematical models developed, especially within the context of culturally relevant applications in students' own lives.

Component	Details
Source	TEKS: Algebra 1
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 2. Reason abstractly and quantitatively. SMP 4. Model with mathematics. SMP 5. Use appropriate tools strategically. SMP 7. Look for and make use of structure. SMP 8. Look for and express regularity in repeated reasoning.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C4: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• Self-Awareness: The ability to recognize one's emotions, thoughts, and values, and how they influence behavior. • Social Awareness: The ability to understand the perspectives of others and empathize with their experiences.
Example(s)	• Students reflect on their own experiences with change and growth, connecting their personal lives to mathematical concepts like slope and rate of change. • Through culturally responsive examples, students explore how slope and rate of change relate to issues in their communities.

Lesson Overview and Goals

This lesson introduces students to **slope** and **slope-intercept** form in real-world contexts. Students will learn to interpret slope as a rate of change, identify the y-intercept as the starting value, and apply these concepts to create and analyze linear equations. Through collaborative activities and culturally relevant examples, students will build a solid understanding of linear relationships. The lesson emphasizes culturally responsive teaching by connecting math to students' lived experiences and promoting positive identity development through asset-based discussions.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C4: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Student outcomes	By the end of the lesson, students will know or be able to define and interpret slope and y-intercept in real-world contexts, construct and graph linear equations, and connect these skills to meaningful, everyday scenarios. They will also collaborate effectively with peers, building social–emotional skills and developing critical thinking in using math for decision-making.
Key vocabulary	Slope (m) as a rate of change, y-intercept (b) as a starting value, slope-intercept form
Prior knowledge	Coordinate plane, x-axis, y-axis, points, graphing slope-intercept form, rise/run, slope, solving for y, positive slope, negative slope
Materials needed	• Graph paper • Whiteboard/marker/eraser • Desmos/graphing calculator • Real-world examples (linked above)

Lesson Procedures

Directions/Script
Launch (10 minutes)
Teacher will ask students to quietly reflect on experiences in their communities that involve a steady change or constant rate. Teacher will then ask students to have a group discussion where they will share out. Encourage them to share their personal stories or observations. • Example prompts: » Increasing cost of certain items in their neighborhood stores » Increasing income or expanding progress in local businesses • Introduce slope concept: » Relate their responses to the concept of slope as the rate of change

Directions/Script
Explore (35 minutes)
Direct Instruction (10 minutes) Explain slope and slope-intercept form using culturally relevant examples. • Examples: » Rate of change in voter registration rates » Rate of change in local businesses or public resources like parks or libraries » Growth of a popular artist's social media following » Showing the relationship between the hours worked and the wage earned » Rate at which a community garden produces food over time • Show how to write a simple equation using slope (m) and y-intercept (b)—starting point in slope-intercept form from the examples above or other examples that you come up with. • Graph examples showing how the differing values of slope change the line. **Guided Practice (10 minutes)** • Graph using paper or Desmos: Give them other real-world examples to come up with the simple equations and then graph them. • In their groups, have students discuss the differences in the slopes and y-intercepts of each problem. Identify why they are different. **Independent Practice (15 minutes)** • Students write equations and graph them from real-world word problems, independently. » Problems should be relevant to the diverse population in your class. » Students need to identify the slope and y-intercept from the scenarios. » Write the equations from that information, and write equations from real-world graphs or tables.
Debrief and closure (10 minutes)
Teacher brings the students back together to reflect on the lesson. • How does slope connect to the changes we see in our communities? • How does this math help us make sense of the world? • How does this math connect to us personally? • Why do we need it?

Lesson C5. Tenet 3: Middle Grades—Ratios and Proportional Relationships

This lesson was created by a teacher during a design lab focused on social and emotional Learning (SEL) and culturally responsive and sustaining education (CRSE). The lesson aligns with Tenet 3, which is described in Table 1.

Lesson C5: Table 1. Overview of Tenet

Component	Details
Essential tenet	#3: Collective Responsibility
Student outcomes	1. Students possess and use interpersonal skills to engage in mathematical discourse and communicate their ideas and findings.
Guideline	1.d. Include real-world mathematical problems that deliberately employ perspective taking in the problem-solving and collaborative activities as a means to build empathy and relationship skills.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C5: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Source: Common Core State Standards
Domain	
Content standards	**7.RP.A.2 & 7.RP.A.3:** Students solve a multistep problem involving ratios and rates. They determine various unit rates of runners and adjust these rates given different distances and times needed for the race. **7.IF.B.6:** Students determine the rate at which the distance between the quarterback and receiver is changing. Finally, students generalize this relationship by examining data and writing an expression that gives the change in distance between the quarterback and the receiver.
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 2. Reason abstractly and quantitatively. SMP 3. Construct viable arguments and critique reasoning of others. SMP 4. Model with mathematics. SMP 8. Look for and express regularity in repeated reasoning.

Tables 3 and 4 list the SEL competencies to which this lesson aligns for both students and teachers.

Lesson C5: Table 3. SEL Competencies for Students

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The ability to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Relationship Skills:** The ability to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
Example(s)	• Students identifying diverse social norms, including unjust ones • Students practicing teamwork and collaborative problem-solving

Lesson C5: Table 4. SEL Competencies for Teachers

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The ability to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Responsible Decision-making:** The ability to make caring and constructive choices about personal behavior and social interactions across diverse situations.
Example(s)	• (Social) awareness of student struggles and challenges • Actively listening to student responses and demonstrating curiosity and open-mindedness

Lesson Overview and Goals

We reason that fair distributions, the concept of ratio, and fractions are about belonging to sets and wholes.

Lesson Setup

Table 5 provides some key information to help teachers set up and carry out the lesson.

Lesson C5: Table 5. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/ resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to determine unit rate by using the distance/time or miles/hour based on given times to determine the average rate.
Key vocabulary	ratios, rates, unit rates, data, writing expressions
Prior knowledge	The YouTube video Ratios and Unit Rate Examples and Word Problems! can be used to clarify misconceptions students have about unit rate and ratios.

Lesson Procedures

Directions/Script
Launch (11 minutes)
Warm-up Activity **Objective:** Introduce unit rates through the lens of high-profile athletes while promoting cultural relevance and collaboration. 1. **Video Viewing (3 minutes)** • Action: Begin by showing a short clip (2–3 minutes) of Sha'Carri Richardson, Julien Alfred, and other athletes racing in the 100-meter dash. • Focus: Encourage students to observe the athletes' techniques and speeds and think about how their cultural backgrounds might influence their running styles or motivations. 2. **Group Discussion (4 minutes)** • Facilitate Discussion: » Ask students to share their initial thoughts on the video. » Questions to consider: › What impressed you about the athletes' performances? › How do their backgrounds contribute to their stories in sports? » Transition into discussing the specific times of Sha'Carri and Julien from the race. 3. **Introduction to Unit Rates (3 minutes)** • Graphical Representation: » Briefly introduce the concept of unit rates by explaining distance/time (e.g., meters per second). » Set up a simple chart on the board: › Example: If Sha'Carri runs 100 meters in 10 seconds, the unit rate is 10 m/s. • Collaborative Prediction: » Divide students into small groups (3–4 students). » Each group predicts outcomes for the 200-meter and 400-meter relay based on the times discussed. Encourage them to consider how factors like stamina and strategy might affect their predictions. 4. **Wrap-up (1 minute)** • Ask a few groups to share their predictions and reasoning. • Highlight the importance of understanding unit rates in analyzing athletic performance and how cultural backgrounds can shape athleticism and competition.

Directions/Script
Explore (35 minutes)

Teacher will facilitate while students use computers or smartphones to find unit rates for each day or each game that athletes perform to make predictions for students to engage in Activity 1 and Activity 2 as time permits.

Activity 1: Team Research and Predictions

Objective: Explore sports data through research and collaboration while connecting to students' cultural backgrounds and interests.

1. **Team Formation:**
   - Break students into diverse teams based on their favorite sports (football, basketball, etc.). Encourage students to share why these teams are significant to them culturally or personally.
2. **Research:**
   - Each team researches their favorite teams online, focusing on
     - » Player stats (e.g., quarterback throws, free throws)
     - » Historical significance or cultural impact of the teams in their communities
3. **Data Collection:**
   - Teams find stats from the first game of the season.
   - Use the unit rate formula to predict performance for the next five games.
   - Encourage discussions of how cultural factors (like regional fan support) might influence team performance.
4. **Group Discussion:**

- Teams present their findings, discussing not only stats but also the cultural significance of their teams. What unique factors influence performance based on their backgrounds?

Activity 2: Data Visualization and Analysis

Objective: Visualize sports data and engage in comparative analysis while fostering teamwork.

1. **Graph Creation:**
   - Students use collected data to create graphs displaying unit rates for each game based on first-game stats. Incorporate various graphing methods (bar graphs, line graphs) to reflect different cultural perspectives on data representation.
2. **Player Comparison:**
   - Each team selects two players with similar stats and discusses their cultural backgrounds or stories. This discussion can highlight how diverse backgrounds contribute to their playing styles.
3. **Equation Development:**
   - Using the formula $y = mx + by = mx + b$, teams create equations based on their selected players' stats.
   - Encourage teams to reflect on how their chosen players' cultural backgrounds may influence their play and performance metrics.
4. **Collaborative Presentation:**
   - Teams present their graphs and equations, discussing the cultural narratives behind their players. This reinforces teamwork and allows for a deeper understanding of the data and its implications.

Directions/Script
Debrief and Closure (10 minutes)

Reflection and Discussion

- **Cultural Relevance:**
  - » Throughout the activities, emphasize how culture shapes athletes' experiences and performance.
  - » Facilitate discussions on how sports can serve as a bridge for understanding diverse cultures.
- **Collaborative Learning:**
  - » Encourage peer feedback and discussion to foster a collaborative environment, allowing students to learn from one another's insights and perspectives.
- **Understanding Proportionality:**
  - » If students track the number of hours spent practicing versus their performance in a game, they might collect data over 5 days. If on each day, they find that more hours correlate with better performance (e.g., time spent practicing is always in the same ratio to points scored), then that data is proportional.
- **Data Collection:**
  - » Students could collect data like
    - › Day 1: 2 hours = 10 points
    - › Day 2: 3 hours = 15 points
    - › Day 3: 4 hours = 20 points
    - › Day 4: 5 hours = 25 points
    - › Day 5: 6 hours = 30 points
  - » Here, the unit rate remains consistent at 5 points per hour, indicating proportionality.

Factors Contributing to Proportional Data

1. **Variables Affecting Performance:**
   - Injuries: A student might perform worse if injured, impacting the ratio of practice to performance.
   - Time in the game: More time might lead to better performance but could also lead to fatigue.
   - Age: Younger students may have different stamina and recovery times compared to older peers.
   - Stamina: If a student has higher stamina, they might sustain performance longer, affecting the ratio over time.
2. **Cultural Relevance:**
   - Incorporating students' backgrounds, such as different training methods or cultural approaches to practice, can also influence outcomes. Discussions around these topics may reveal different interpretations of what success looks like.

Directions/Script

Predictability of Data

1. **Exact Versus Predictable:**
   - Data may be predictable in the sense that certain trends can be anticipated (e.g., more practice generally leads to better scores), but it isn't exact due to various external factors.
   - For instance, even if Sha'Carri Richardson wins a race by a slim margin (like .03 seconds), other elements such as training methods, breathing techniques, mental focus, and nutritional habits could all impact performance.
2. **Complex Interactions:**
   - Performance is rarely based on one factor. A well-rounded analysis would consider the following:
     - » How consistent training impacts performance over time
     - » The role of psychological readiness on race day
     - » The importance of recovery and rest, especially after intense training

Student Collaboration

- **Group Discussions:**
  - » Students can collaborate to explore these factors, perhaps working in small groups to analyze their own data and the experiences of athletes. They could discuss questions like: "How does our culture shape our approach to practice?" or "What other variables should we consider in our data analysis?"

Elevating SEL in the Lesson

In the Explore Section of this lesson, students work collaboratively and draw on the following SEL competencies:

- » Self-awareness for understanding and articulating their thinking and for identification of feelings, strengths, and challenges
- » Responsible decision-making for perseverance in problem-solving
- » Relationship skills necessary for positive and harmonious engagement with peers in group discussions

Lesson C6. Tenet 2: Middle Grades—Functions

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 2, which is described in Table 1.

Lesson C6: Table 1. Overview of Tenet

Essential tenet	# 2: Agency and Belonging
Student outcomes	1. Students demonstrate mathematical confidence and agency through their participation in classroom discussion and activities.
Guideline	Support collaborative, peer-to-peer conversations that invite curiosity and inquiry, build collective understanding, and offer a judgment-free space where students feel comfortable offering ideas and answers they aren't sure about.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C6: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	
Content standards	**8.F.A.1:** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.A.2:** Compare properties of two functions each represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). **8.F.A.3:** Interpret the equation y = mx + by = mx + by = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. **8.F.B.4:** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship, a table, or a graph, and interpret these in terms of the situation.

Component	Details
Source	Common Core State Standards
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 2. Reason abstractly and quantitatively. SMP 3. Construct viable arguments and critique the reasoning of others. SMP 4. Model with mathematics. SMP 5. Use appropriate tools strategically. SMP 7. Look for and make use of structure.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C6: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Self-Awareness:** The ability to understand one's own emotions, thoughts, and values and how they influence behavior across contexts. This includes capacities to recognize one's strengths and limitations with a well-grounded sense of confidence and purpose.
Example(s)	Actively listening to peers during group work/discussions provides students opportunity to learn and practice how to respect different perspectives and provide constructive feedback on their partner's reasoning. This will foster a supportive classroom environment where students feel valued and respected.

Lesson Overview and Goals

In this lesson, students will explore the differences between linear and nonlinear functions by analyzing graphs, equations, and tables. They will work in groups to compare key characteristics such as the rate of change, graph shape, and structure of equations

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C6: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	Students will be able to identify the differences between linear and nonlinear functions by analyzing graphs and discussing key characteristics in groups.
Key vocabulary	Linear, nonlinear, constant rate of change
Prior knowledge	Understanding variables and expression, basic concept of function, and rates of change and plotting on coordinate plane, creating input-output tables, and identifying patterns

Lesson Procedures

Directions/Script
Launch (5–10 minutes)
Warm-up Activity: Analyzing Graphs The goal of this warm-up is for students to observe and describe the features of graphs using their own words. Throughout the lesson, students will encounter various graphs and progressively develop more precise terminology to describe them. **Instructions:** 1. **Group Setup:** Arrange students in groups of two to four. 2. **Display the Graphs:** Show an image with four different graphs for the class to see. 3. **Group Discussion:** Allow students time to share and discuss their ideas within their groups. **Whole-class Discussion:** 1. **Group Sharing:** Invite each group to share one reason why a particular graph might not belong. 2. **Class Feedback:** Record and display the responses for everyone to see. After each response, ask the class if they agree or disagree. 3. **Encourage Explanation:** Since there is no single correct answer, focus on students' reasoning and ensure their explanations are logical and accurate. This activity encourages critical thinking, collaboration, and the development of precise mathematical language.

Directions/Script
Here are some questions a teacher can ask during this warm-up to guide students' thinking and discussion: **During Individual Thinking Time:** • "What features of the graphs stand out to you?" • "Can you notice something unique about each graph?" • "What might make one graph different from the others?" **While Circulating During Group Discussion:** • "What have you noticed about the graphs so far?" • "Does everyone in your group agree? Why or why not?" • "Can you think of more than one reason why a graph might not belong?" • "How would you explain your choice to someone outside your group?" **During the Whole-class Discussion:** • "Which graph did your group choose, and why?" • "Does anyone else have a different reason for why this graph might not belong?" • "Can anyone build on or challenge this reasoning?" • "What patterns or similarities do you see among the other graphs?" • "How are the graphs that belong alike?" • "Is it possible for multiple graphs not to belong? Why?" **Encouraging Reflection:** • "How did your thinking change after hearing your classmates' ideas?" • "What new words or phrases did you use to describe the graphs today?" • "What else do you wonder about these graphs?" These questions aim to foster curiosity, critical thinking, and meaningful collaboration while encouraging students to listen to and value diverse perspectives in a respectful and supportive environment.

Directions/Script
Explore (30 minutes)

Display the slide with table.

Students will read the prompt and the table that shows the approximate height and horizontal distance traveled by a football kicked at an angle of 30° with an initial velocity of 30 yards per second. Students will discuss whether the rate of change for the height or the distance traveled is constant or not.

Before Students Begin:

- "What information does the prompt and table provide?"
- "What do you notice about the relationship between height and distance in the table?"
- "What do you predict about how the height or distance might change as the football travels?"

While Students Analyze the Data:

- "What happens to the height of the football as the horizontal distance increases? How can you describe this change?"
- "Does the height increase or decrease at a constant rate? How can you tell?"
- "What do you notice about the horizontal distance traveled? Is the rate of change consistent?"
- "What patterns or trends do you see in the table?"
- "How does the data in the table reflect the motion of the football?"

During Group Discussions:

- "What evidence can you use to support whether the rate of change is constant or not?"
- "How would you explain your reasoning to someone in your group?"
- "Do all members of your group agree? Why or why not?"
- "Can you compare the height and horizontal distance? How are their rates of change different or similar?"
- "What real-world factors might influence these rates of change?"

For Whole-class Reflection:

- "Who thinks the rate of change for the height is constant? Why or why not?"
- "What about the horizontal distance—does it change at a constant rate? How do you know?"
- "How do the height and distance reflect the path of the football?"
- "How might this table help us understand other real-world examples, like throwing a ball or launching a rocket?"

Encouraging Deeper Thinking:

- "How would the table look if the initial velocity or angle changed?"
- "What kind of graph might represent the data in this table?"
- "What questions do you still have about the relationship between height and distance in this scenario?"

Directions/Script
Display the slide with the graph. Using the same table, now graph the ordered pairs (time, height) and (time, distance) on separate coordinate graphs. Connect the points with a straight line or smooth curve. Then compare the graphs. 1. **Graphing Activity:** » Have students graph the ordered pairs (time, height) on one coordinate plane. » On a separate graph, plot the ordered pairs (time, distance). » Students should connect the points using either a straight line (if the rate of change is constant) or a smooth curve (if it varies). 2. **Comparison Activity:** » Students compare the shape and patterns of the two graphs. **Teacher Questions for Graphing:** • "What do the ordered pairs (time, height) (time, distance) represent in this situation?" • "How can we determine the best way to connect the points—a straight line or a smooth curve?" • "What does the shape of the graph tell us about how the football moves over time?" • "How do the axes on the graph help us understand what is happening to the height and distance at different times?" **Teacher Questions for Comparison:** • "How are the two graphs similar? How are they different?" • "What does the graph of (time, height) tell us about how the height changes over time?" • "Does the graph (time, distance) show a constant or varying rate of change? How do you know?" • "Which part of the graph shows when the football is at its highest point? What does that look like in the table?" • "What might explain the difference in the shapes of the two graphs?" • "How does the graph (time, height) relate to the football's motion in the air?"

Directions/Script
Debrief and Closure (5–10 minutes)
"In your own words, compare linear and nonlinear functions based on what you learned in today's lesson. Consider their graphs, rates of change, and any patterns you noticed." Success Criteria for the Exit Ticket (shared with students): 1. **Graphs:** Describe how the graphs of linear and nonlinear functions look different (e.g., straight lines vs. curves). 2. **Rate of Change:** Explain whether the rate of change is constant or not for each type of function. 3. **Patterns:** Share any specific patterns or trends you observed in the graphs or tables. 4. **Connection to the Activity:** Relate your explanation to the graphs and data you worked with during the lesson (e.g., time vs. height was nonlinear, time vs. distance might have been linear). Sample Student Responses (to model expectations): • Linear functions have graphs that are straight lines, and the rate of change is the same everywhere. For example, the distance over time might increase by the same amount each second. • Nonlinear functions have graphs that are curved, like the height of the football, which increased and then decreased. The rate of change isn't constant because the height changes faster at some points than others. • In today's lesson, the graph of time versus distance looked linear, while the graph of time versus height was nonlinear because the height reached its peak and then came back down.

Lesson C7. Tenet 2: Elementary Grades—Use Division to Solve Problems

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 2, which is described in Table 1.

Lesson C7: Table 1. Overview of Tenet

Essential tenet	# 2: Agency and Belonging
Student outcomes	2. Students develop and exhibit their intrinsic motivation and emotions toward mathematical learning, which is evident in their engagement levels, task endurance, and ability to sustain problem-solving stamina. 5. Students feel safe to engage in productive struggle, evidenced by their willingness to take risks, feedback, and revise their thinking. This includes students sharing unfinished thinking and celebrating their progress and peer's work.
Guidelines	(c) Support collaborative, peer-to-peer conversations that promote academic scaffolds, safe spaces for conversation, and organic collective inquiry. (i) Prompt students to reflect on the complex emotions involved in mathematics success, such as intellectual humility and vulnerability and how emotions affect one's learning experiences.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C7: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	TEKS 4.4.E
Domain	
Content standard	Use division to solve problems (no more than one-digit divisors and three-digit dividends) using the algorithm.
Standards for Mathematical Practice (SMP)	Construct viable arguments and critique the reasoning of others.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C7: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Responsible Decision-Making:** The abilities to make caring and constructive choices about personal behavior and social interactions across diverse situations. • **Self-Management:** The ability to manage one's emotions, thoughts, and behaviors effectively in different situations and achieve goals and aspirations.
Example(s)	• Students proactively use metacognitive strategies and can justify their mathematical understandings and thoroughly describe their decision-making. • Students seek assistance and resources in service of self-management.

Lesson Overview and Goals

Students will be able to engage in effective and positive discussion that evaluates how their peers completed a math problem.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C7: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to engage in effective and positive discussion that evaluates how their peers completed a math problem.
Key vocabulary	None (students will discover any new words needed)
Prior knowledge	Problem-solving

Lesson Procedures

Directions/Script
Launch (10 minutes)
• Tell the students that today they're going to learn a strategy for solving long division problems. • Write the word "division" on the board. Let the students know that division is an operation that tells us the number of groups that can be made out of another number. • Students will discuss what they understand about division. • Ask students to brainstorm aloud why it is important to use accountable talk when discussing or disagreeing with a peer. • Explain that when we engage in discussion, we may not always agree, so it is important to have strategies for engaging in communication effectively.
Accountable Talk Posters or Math Talk Cards
Using the Long Division Vocabulary Sheet, introduce vocabulary words.
Explore (30–40 minutes)
Write a problem on the board. • Tell the students to pick up their supplies and write the problem down. • Ask for volunteers to raise their hands and share with the class the steps needed to complete this problem. • Using the accountable talk cards, have students discuss what they are hearing/thinking about the problem. • Give the students time to solve the problems. • Ask the students to dictate to you how they solved the problem. • Write their responses on the board. • Review the problem with the class to check for accuracy.
Group Practice: • Divide students into groups. • Pass out the Long Division w/ Decimals problem cards and poster paper. • Student groups will solve ALL problems. The teacher will assign one problem for them to present. • Students will illustrate their division problems on poster paper/butcher paper. • Teacher will walk around the room to monitor the groups' discussions and assist students as needed.

Directions/Script
Gallery Walk: • Assign each group one problem to illustrate and share. • Ask each group to share their answer to the problem and how they got to that answer. • Encourage students to compare and contrast their own processes using the accountable talk sentence stems. • As the problems are reviewed during the gallery walk, students will check their answers for accuracy. • After each problem has been checked for accuracy, ask the class if there are any questions or comments about long division. • Ask for volunteers to share their experiences. Ask students to explain what they learned from this activity.
Debrief and Closure (10 minutes)
Ask: • "What are you most proud of from your group's work?" • "How were you able to help others in your group?" • "How do you feel about your problem-solving skills today?"
Long Division Exit Ticket

Elevating SEL in the Lesson

Students work collaboratively and draw on the following SEL competencies:

- » Self-awareness for understanding and articulating their thinking and identification of feelings, strengths, and challenges
- » Responsible decision-making for perseverance in problem-solving and help seeking relationship skills necessary for positive and harmonious engagement with peers in group discussions

Lesson C8. Tenet 2: Elementary Grades—Multiply Multidigit Numbers

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 2, which is described in Table 1.

Lesson C8: Table 1. Overview of Tenet

Essential tenet	# 2: Agency and Belonging
Student outcomes	3. Students proactively use metacognitive strategies and can justify their mathematical understanding and thoroughly describe their decision-making.
Guideline	Encourage students to use metacognition to thoroughly describe their problem-solving process and that guide reflection on decisions or revisions made to reach a solution.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C8: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core; Illustrative Math
Domain	
Content standard	**5.NBT.B.5:** Fluently multiply multidigit whole numbers and respond to the mathematical thinking of others.
Standards for Mathematical Practice (SMP)	SMP 3. Construct viable arguments and critique the reasoning of others. SMP 7. Look for and make use of structure.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C8: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Responsible Decision-Making:** The abilities to make caring and constructive choices about personal behavior and social interactions across diverse situations. • **Social Awareness:** The abilities to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts.
Example(s)	• Students use metacognitive strategies and can justify their mathematical understandings and thoroughly describe their decision-making. • Students take on others' perspectives and recognize strengths in how others approached solving the problems.

Lesson Overview and Goals

In this lesson, students have the opportunity to multiply using their own strategy. The teacher serves as a facilitator allowing students to select their own approach to multiplication. Students discuss possible strategies in a number talk. Students then use the strategies as they work out problems they have selected in their workbooks.

There are a variety of problems for students to choose from—ones with single-digit factors and others with multipledigit factors. Teachers should encourage students to use different strategies to support rich conversation during student sharing.

At the end of the lesson, students reflect on their approach to multiplication, challenges they faced, and something interesting they learned about multiplication. This lesson supports students in developing a sense of belonging in their strategy and agency in their ability to choose their own mathematical approach.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C8: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to • orally explain their mathematical approach in solving a multiplication problem, and • respond to the mathematical thinking of others.
Prior knowledge	• Students understand that there are multiple ways to solve multiplication problems. • Students are familiar with representing multiplication visually. • Students are familiar with multiplication as repeated addition.

Lesson Procedures

Directions/Script
Launch (10 minutes)
Teacher presents learning objectives and SMP of focus using SMP math posters.
Teacher facilitates a number talk that has opportunities for students to flexibly solve for the product mentally as the factor increases. 1. Find the value of each expression mentally: 1. 230 × 10 2. 230 × 12 3. 230 × 15 4. 232 × 15 **Teacher questions to facilitate the discussion:** • "Does this match what you were thinking?" • "Can we name and define this strategy?" • "Are there helpful patterns in this number string?" • "Are there any problems that we solved in the same way/differently?" • "Which problem was the most challenging to solve mentally?"

Directions/Script
Explore (45 minutes)
Teacher launches Task 1: Choose a Multiplication Strategy. • Say: "Look at the problems. Choose two problems you would solve using a different strategy." Students will then describe their approach in solving the problem to a partner. a. 14 × 3 b. 14 × 101 c. 14 × 25 d. 14 × 9 e. 14 × 136 Allow independent work time to solve the value of each expression (5–10 minutes). Then, allow time for partner work (8–10 minutes). Circulate around the classroom and select two students who solved the same problem but used a different approach. Lead a discussion using the strategy Compare and Connect in which students share their mathematical approach to solving a problem. Teacher questions to facilitate the discussion: • "What is the same/different between these strategies?" • "Why did the different approaches lead to the same outcome?" • "Are there any benefits or drawbacks to one strategy compared to the other?" ***MLR7 Compare and Connect (Source: Illustrative Math)***
This routine fosters students' meta-awareness as they identify, compare, and contrast different mathematical approaches and representations. Students are prompted to reflect on, and linguistically respond to, these comparisons: for example, exploring why or when one might do or say something a certain way or by identifying and explaining correspondences between different mathematical representations or methods. How it happens: 1. **Students prepare displays of their work:** Students are given a problem that can be approached and solved using multiple strategies or a situation that can be modeled using multiple representations. Students prepare a visual display of their work, paying attention to the language and details they include that will allow others to make sense of their approach and reasoning. 2. **Compare:** Students investigate each other's work, pointing out important mathematical features, and making comparisons. Comparisons should focus on the typical structures, purposes, and affordances of the different approaches or representations: what worked well in this or that approach or what is especially clear in this or that representation. 3. **Connect:** Students identify correspondences in how specific mathematical relationships, operations, quantities, or values appear in each approach or representation. During the discussion, amplify language students use to communicate about mathematical features that are important for solving the problem or modeling the situation. Call attention to the similarities and differences between the ways those features appear.

Directions/Script
Teacher launches Task 2: Compare Strategies. 1. Say: "In this task you will first solve for the product independently. Then, you will have some time to explain your mathematical approach and compare it to the others in your small group." 2. Present sentence stems for students to share their mathematical approaches and make connections. a. I solved _____ by _____. b. My strategy for the equation _____ was _____. c. Something similar that I did was _____. d. We both _____.
Debrief and Closure (5 minutes)
1. Teacher synthesizes the lesson by asking students to reflect on multiplication. a. Describe a challenge with multiplication. b. Describe your approach to multiplication. Does it always work? If not, why? c. Describe something new or interesting you learned about multiplication.

Lesson C9. Tenet 1: Geometry—Using Graphs to Represent Problems

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 1, Identity Affirming, which is described in the following Table 1.

Lesson C9: Table 1. Overview of Tenet

Essential tenet	#1: Identity Affirming
Student outcomes	2. Students demonstrate a deep understanding of everyday mathematics inside their homes, schools, and communities; within their cultural backgrounds; and across their interests.
Guideline	Invite students to make connections between classroom learning and their valuable knowledge and experience from outside the classroom.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C9: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	Geometry
Content standard	**5.G.A.2:** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Standards for Mathematical Practice (SMP)	SMP 2. Reason abstractly and quantitatively. SMP 7. Look for and make use of structure.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C9: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The abilities to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. ognizing and appreciating diverse cultural perspectives.
Example(s)	• Students take on others' perspectives.

Lesson Overview and Goals

The task was adapted from Illustrative Mathematics Grade 5, Unit 7, Lesson 12, and utilizes the lesson planning template from Stride 3: *A Pathway to Equitable Math Instruction: Creating Conditions to Thrive* (pages 13–14).

The lesson is intended to do the following:

» Support development of mathematical identity by fostering a positive learning environment where students see themselves as mathematicians in a math community where discourse supports diverse perspectives and active listening.

» Create opportunities for students to discern structure when representing points on the coordinate grid.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C9: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to • represent problems on the coordinate grid, and • describe the location of a point on a coordinate grid using mathematical vocabulary.
Key vocabulary	None (students will discover any new words needed.)

Aspect of lesson setup	Details
Prior knowledge	**Number lines:** Students should be familiar with plotting points on a number line. This includes understanding the concept of a number line as a visual representation of numbers and their order. **Perpendicular lines:** They should understand what perpendicular lines are and how they intersect. This helps in constructing the x and y-axes of the coordinate plane. **Ordered pairs:** Students need to be comfortable with the concept of ordered pairs (x, y) and understand that the first number represents the x-coordinate (horizontal position) and the second number represents the y-coordinate (vertical position). **Coordinate plane:** They need to understand the structure of the coordinate plane, including the x-axis and y-axis and the origin (0,0). **Graphing points:** Students should know how to plot points on a coordinate grid, moving horizontally along the x-axis first and then vertically along the y-axis.

Lesson Procedures

Directions/Script
Launch (10 minutes)
Teacher presents learning objective and SMP of focus using SMP math posters.
Teacher demonstrates a variety of coins including currency of the United States, Mexico, El Salvador, and Colombia. Teacher engages students to share their ideas in the routine "What do you know about __?" 1. Have students work in groups of two. 2. Display the question. 3. Ask, "What do you know about coins?" 4. Provide 1 minute of quiet think time. 5. Allow 2 minutes of partner talk time. 6. Group share and record responses. 7. Encourage students to see that the coins are from different places and to share if they are familiar with coins from additional places around the world. You can even encourage students to bring in coins that they may have from their home country or places they have visited in advance of this lesson.

Directions/Script
Explore (40 minutes)
Teacher launches Task 1: Heads or Tails using the strategy Three Reads with the following prompts. 1. **Read #1: "What is the situation about?"** After a shared reading, students describe the context. 2. **Read #2: "What can be counted or measured in this situation?"** After the second read, students list all the quantities. 3. **Read #3: "What questions can we ask about this data?"** Students discuss possible questions. Note: The problem will be adjusted to include names of students in the class. The problem will be presented on an anchor chart poster.
Students work on answering questions generated with a partner. Students also work on adding their own coordinates to the grid. After partners have had some time to answer the questions and graph their data, students will engage in a whole-class discussion. The teacher will call on some students to plot their data on the class anchor chart. Sentence stems for students to share their data: 1. The point that represents my flips is (_____ , _____). 2. I/My partner graphed the point (_____ , _____). Sentence stems for classmates to engage in the data presented by their peers: 1. (_____ , _____) represents. … 2. If _____ plotted (_____ , _____) that means. …
Teacher launches Task 2: Coin Values using the strategy See, Think, Wonder.
Students then work with partners to answer questions about the data. 1. Tyler has 1 dime, 3 nickels, and 2 pennies. Which point represents Tyler's coins? 2. Lin has 3 quarters, 1 dime, and 1 penny. Which point represents Lin's coins? 3. Diego has 1 quarter and 1 dime. Write the coordinates that represent Diego's coins. 4. Clare has 5 coins and does not have a quarter. Write the coordinates of the point that represents Clare's coins. What coins might she have?
Debrief and Closure (10 minutes)
Teacher synthesizes the activity by calling on volunteers to attend to the learning goal. • Say, "Today we represented data on the coordinate grid. Our learning goal was to describe the location of a point on the coordinate grid. Describe to your partner how the coordinate grid works. How can you explain the location of a point on the coordinate grid?"
Students will complete the exit ticket and evaluate themselves on the learning goal by submitting their exit ticket on a self-monitoring scale folder of 1–4.

Lesson C10. Tenet 2: Elementary Grades—Numbers and Operations

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns to Tenet 2, which is described in Table 1.

Lesson C10: Table 1. Overview of Tenet

Essential tenet	# 2: Agency and Belonging
Student outcomes	1. Students proactively use metacognitive strategies and can justify their mathematical understandings and thoroughly describe their decision-making. 5. Students feel safe to engage in productive struggle, evidenced by their willingness to take risks, make errors, receive feedback, and revise their thinking. This includes students sharing their unfinished thinking and celebrating their progress and peers' work.
Guidelines	b. Encourage students to use metacognition to thoroughly describe their problem-solving process and that guide reflection on decisions or revisions made to reach a solution. (self-awareness, self-management, responsible decision-making) c. Support collaborative, peer-to-peer conversations that promote academic scaffolds, safe spaces for conversation, and organic collective inquiry. (self-awareness, self-management, responsible decision-making) g. Incorporate reflective practices, self-assessment tools, and cycles for students to solidify learning and make connections to new mathematical learning. (self-awareness, self-management, responsible decision-making)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C10: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	CCSS
Domain	Numbers and Operations
Content standard	**2. NBT.A.1:** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
Standard for Mathematical Practice (SMP)	SMP 5. Use appropriate tools strategically.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C10: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The abilities to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Responsible Decision-Making:** The abilities to make caring and constructive choices about personal behavior and social interactions across diverse situations.
Example(s)	• Students proactively use metacognitive strategies and can justify their mathematical understandings and thoroughly describe their decision-making. • Students use critical thinking to determine what tool(s) they should use to represent three-digit numbers in different ways.

Lesson Overview and Goals

The lesson is intended to do the following:

- » Connect to the SEAD theme of agency.
- » Allow students to reflect on the different math tools they have at their disposal and determine which tool will be most useful to represent three-digit numbers.
- » Promote student agency by establishing the teacher as a facilitator, posing the problem and supporting students to determine what tools can support them in their task.
- » Engage students in the Standards for Mathematical Practice (SMP) 5: Use appropriate tools strategically.

Objective of lesson:

I can pick the right tools to represent three-digit numbers in different ways.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C10: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Math Slides
Student outcomes	By the end of the lesson, students will know or be able to pick the right tools to represent three-digit numbers in different ways.
Key vocabulary	Place value, digit, hundreds, tens, ones
Prior knowledge	• Place value of two-digit numbers • Use of math tools
Materials	• Slides • Math manipulatives: base 10 blocks/100s charts/unifix cubes/2-color counters/number lines • Paper and pencils

Lesson Procedures

Directions/Script
Launch (5–7 minutes)
1. Brainstorm with students around the type of tools they use in mathematics. What tools do we use? Why do we use them? Do some tools work better than others for specific math tasks? 2. Teacher sets the stage for learning by introducing the learning target and success criteria.

Directions/Script
Explore (25 minutes)
3. Discuss with students about what it means to use math tools. 4. Say the number "253" to students. Tell them that they are going to use their tools to represent this number in as many different ways as they can think of. It is up to them to choose the different tools they will use to represent the number. Some possible representations will include writing the number with numerals, using base 10 blocks to model the number and drawing a picture of a base 20 model on a whiteboard or paper. Some tools will be excellent for representing such a large number, and some tools will be inefficient. It is up to the students to determine which tools will be best. 5. Give students time to work on their own to represent the given number in different ways. Walk around the room checking in with students as they work. Some questions to pose to students: » Why did you choose this tool to help you represent the number? » What are some tools that you thought wouldn't be good for representing this number? Why? 6. While you are walking around the room, look for student samples of different ways to represent the number. Make note of which students you will be asking to share their work with the class.
Debrief and Closure (5–7 minutes)
7. Back in the whole group, ask students to share their strategies. Ask them why they chose the tools they chose. Discuss with the class about which tools they thought were not good for representing the number and why. 8. Did we meet our success criteria? How do we know?

Lesson C11. Tenet 2: Elementary Grades—Numbers and Operations

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 2, which is described in Table 1.

Lesson C11: Table 1. Overview of Tenet

Essential tenet	#2: Agency and Belonging
Student outcomes	2. Students develop and exhibit their intrinsic motivation and emotions toward mathematical learning, which is evident in their engagement levels, task endurance, and ability to sustain problem-solving stamina. 5. Students feel safe to engage in productive struggle, evidenced by their willingness to take risks, feedback, and revise their thinking. This includes students sharing unfinished thinking and celebrating their progress and peer's work.
Guideline	(c) Support collaborative, peer-to-peer conversations that promote academic scaffolds, safe spaces for conversation, and organic collective inquiry. (i) Prompt students to reflect on the complex emotions involved in mathematics success, such as intellectual humility and vulnerability and how emotions affect one's learning experiences.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C11: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	5th Grade Math TEKS
Domain	
Content standard	**5.3. Number and operations:** The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy.
Standard for Mathematical Practice (SMP)	SMP 3. Construct viable arguments and critique the reasoning of others.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C11: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Responsible Decision-Making:** The abilities to make caring and constructive choices about personal behavior and social interactions across diverse situations. • **Self-Management:** The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
Examples	• Students proactively use metacognitive strategies and can justify their mathematical understandings and thoroughly describe their decision-making. • Student seek assistance and resources in service of self-management.

Lesson Overview and Goals

Students will be able to engage in effective and positive discussion that evaluates how their peers completed a math problem.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C11: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to engage in effective and positive discussion that evaluates how their peers completed a math problem.
Key vocabulary	None (students will discover any new words needed as problems are needed)
Prior knowledge	Problem-solving

Lesson Procedures

Directions/Script
Launch (10 minutes)
These slides were made to be a template for future lessons. The teacher can change out the opening and the word problems. The teacher will choose a day to work on problem-solving skills. By the end of the year, students will know and understand the format, so more time is spent on problem-solving and collaborating and less time on expectations and learning the format.
Slides 2–4 "Would you rather?" If more slides are needed, please go to the end of slides, "EXTRA SLIDES"
• Teacher will show slides with two choices. • Students will choose an option and discuss why they chose that option.
• Ask students to brainstorm aloud why it is important to use accountable talk when discussing or disagreeing with a peer. • Explain that when we engage in discussion, we may not always agree, so it is important to have strategies for engaging in communication effectively.
Accountable Talk Posters or Math Talk Cards
Explore (30–40 minutes)
As a whole class the teacher will walk the students through the example problem. Introduce Problem Solving Table Mat The teacher can print the mats so students can write on them, or the teacher can laminate a class set so that students can use them over and over again. Alternatively, students can divide a whiteboard in four parts to solve the problem. "T" stands for teacher "S" stands for student
Slides 9–11 T - "What information do we need to know?" S - "336 planes take off per day during the Christmas holiday." S - "The airport is open 12 hours each day during the Christmas holiday." T - "Is there any extra information? Information we don't need?" S - "280 planes take off per hour on a normal day."

Directions/Script
Slides 12–13 T - "What is the problem asking us for? Where is the question?" S - "How many planes take off from this airport in each hour?"
Slides 14–15 T - "What operation do you think we will need? And why?" S - "Division" *Answer will vary*.
Slide 16 T - "Show your division on your mat/board. Pencils/markers down when you are done." When 90 percent of the students are finished, solve the problem on the board. Have students tell you each step.
Slides 21–22 *Answers will vary*. What it looks like ... • Getting off task and talking about something not involving the word problem, and then gently reminding the group about the current topic. • Making sure no one is sitting quietly and not contributing • Setting a timer or read the clock. • Adding to the group discussion • Sharing ideas within the group • Understanding the problem and the answer What it doesn't look like ... • Yelling at group members • Starting off topic discussions • Forgetting the time

Directions/Script
Slides 23–24 *Answers will vary*. What it looks like ... • Presenting the final answer • Getting the teacher when help is needed (after the whole group has tried) • Adding to the group discussion • Sharing ideas within the group • Understanding the problem and the answer What it doesn't look like ... • Yelling at group members • Getting help when the group has not tried
Slides 25–26 *Answers will vary*. What it looks like ... • Getting and returns all materials • Making sure that materials are used correctly • Making sure items are not lost • Adding to the group discussion • Sharing ideas within the group • Understanding the problem and the answer What it doesn't look like ... • Yelling at group members • Putting materials up incorrectly

Directions/Script
Slides 27–28 *Answers will vary*. What it looks like … • Taking good notes. • Writing neatly on the final draft for the group • Adding to the group discussion • Sharing ideas within the group • Understanding the problem and the answer What it doesn't look like … • Yelling at group members • Starting off topic discussions • Writing is messy • Not writing what the group decides
Word Problems #1 Word Problems #2 The teacher will read the word problem(s) to the class. The class will be divided into small groups.
Assign groups and jobs.
Students will illustrate their math problem on poster paper/butcher paper. Teacher will walk the room to monitor the groups' discussions. • Ask each group to share their answer to the problem and how they got to that answer. • Encourage students to compare and contrast their own processes using the accountable talk sentence stems. • Ask for volunteers to share their experiences. Ask students to explain what they learned from this activity.
Debrief and Closure (10 minutes)
Ask: • What are you most proud of from your group's work? • How were you able to help others in your group? • How do you feel about your problem-solving skills today?
Exit Ticket: Teachers can have the exit ticket printed out or teach students how to write on a sticky note.

Elevating SEL in the Lesson

Students work collaboratively and draw on the following SEL competencies:

- Self-awareness for understanding and articulating their thinking and for identification of feelings, strengths, and challenges
- Responsible decision-making for perseverance in problem-solving and help seeking
- Relationship skills necessary for positive and harmonious engagement with peers in group discussions

Lesson C12. Tenet 3: Statistics—Analyzing Data

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 3, which is described in Table 1.

Lesson C12: Table 1. Overview of Tenet

Essential tenet	#3: Collective Responsibility
Student outcomes	4. Students are co-creators of learning and shared space in their mathematics classroom and have opportunities to meaningfully contribute and to build on the idea of others with a shared focus on collective responsibility.
Guideline	(s.) Promote co-constructed learning experiences with peers, such as recognizing errors within peers' problem-solving process and supporting corrective actions.

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C12: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Source: Florida's BEST Standards for Mathematics
Domain	Statistics and Probability
Content standards	**MA.7.DP.1.2:** Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers. **MA.7.DP.1.3:** Given categorical data from a random sample, use proportional relationships to make predictions about a population.
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 4. Model with mathematics. SMP 5. Use appropriate tools strategically.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C12: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Responsible Decision-Making:** The abilities to make caring and constructive choices about personal behavior and social interactions across diverse situations. • **Self-Management:** The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
Example	Data collection to obtain results

Lesson Overview and Goals

1. **Math Objective:** Students will learn to collect, analyze, and interpret data using statistical measures such as mean, median, mode, and range.
2. **SEL Objective:** Students will develop responsible decision-making and empathy by understanding how data can reflect the needs and preferences of their school community.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C12: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	This is a follow-up lesson after the students have been taught the standard. The actual project will take a few days to gather the data.
Student outcomes	Students will learn to collect, analyze, and interpret data using statistical measures such as mean, median, mode, and range. Students will develop responsible decision-making and empathy by understanding how data can reflect the needs and preferences of their school community.
Key vocabulary	Biased, unbiased, sample, population
Prior knowledge	Statistics unit

Lesson Procedures

Directions/Script
Launch (10 minutes)
Discuss the Importance of Data: Begin by explaining how data is used to make informed decisions in schools, communities, and businesses. Give examples like using survey data to decide what food should be served in the cafeteria or how companies use customer feedback to improve products. **Class Discussion:** • Ask students to brainstorm areas in school life that could be improved by gathering and analyzing data (e.g., improving the quality of school lunches, reducing cafeteria waste, or making school events more inclusive). • Explain how data helps us understand problems, assess situations, and make decisions that benefit everyone.
Explore (40 minutes)
Data Collection and Analysis **Step 1: Choosing a School-Related Issue:** • Divide students into small groups. Each group will choose a school-related issue to investigate (examples: school lunch preferences, time spent on homework, trash disposal trends, or opinions on school events). **Step 2: Data Collection Methods:** • Groups can either collect their own data by designing a simple survey to ask classmates (e.g., "What is your favorite lunch option?") or work with provided data sets (e.g., previous surveys or data on school waste). • Encourage students to think about how to design their questions to get useful information (e.g., using categories for preferences or asking for numerical values like time spent on activities). **Step 3: Data Analysis:** • Once the data is collected, students will calculate: » **Mean (Average):** What is the average response for their survey question? » **Median:** What is the middle value of their data set? » **Mode:** Which response or category occurred the most often? » **Range:** What is the difference between the highest and lowest values? **Step 4: Creating Graphs:** • Students will create a visual representation of their findings using graphs. They can choose between bar graphs, pie charts, or line graphs, depending on what best displays their data. **Example:** A group investigating school lunch preferences could create a bar graph showing how many students prefer each lunch option. A group analyzing the time students spend on homework could create a line graph showing the range of time spent by different students.

Directions/Script
Debrief and closure (10 minutes)
Writing Reflection: After the analysis, ask students to reflect on what their data revealed about the preferences or needs of their peers. Have them respond to the following prompts in a short paragraph: • How did collecting and analyzing this data help you understand your classmates' opinions and needs? • How could this information be used to make decisions that improve school life for everyone? • How does data help us make fair and informed choices? **Class Discussion:** • Discuss how understanding the needs and opinions of others through data can foster empathy. How can analyzing data be a tool for promoting positive change? • Encourage students to think about how the solutions they propose can be inclusive and benefit the entire school community. • Talk about biases, how they happen, and how they affect the data.

Lesson C13. Tenet 3: Algebra 2—Data Graphing

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 3, which is described in Table 1.

Lesson C13: Table 1. Overview of Tenet

Essential tenet	#3: Collective Responsibility
Student outcomes	Students are partners in creating knowledge and experiences in the mathematics classroom and have opportunities to meaningfully contribute to, and build on, the ideas of others.
Guidelines	Include tasks that give students opportunities to formulate, communicate, and critique arguments with peers and correct misunderstanding in a way that maintains positive relationships. (self-awareness, self-management, social awareness, relationship skills, responsible decision-making)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C13: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	Building Functions
Content standard	**HSF-BF.B.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Standards for Mathematical Practice (SMP)	SMP 3. Construct viable arguments and critique the reasoning of others. SMP 6. Attend to precision.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C13: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	**Relationship Skills:** The abilities to establish and maintain healthy and supportive relationships and to effectively naavigate settings with diverse individuals and groups.
Example(s)	• Communicating effectively • Practicing teamwork and collaborative problem-solving • Showing leadership in groups • Seeking or offering support and help when needed

Lesson Overview and Goals

Learning Goals: Describe informally (orally and in writing) transformations of graphs. Describe (in writing) how to transform a given function to fit a data set.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C13: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will be able to describe how a graph is transformed.
Key vocabulary	translate, reflect, stretch, vertical, horizontal, x-intercept, y-intercept, degree, relative minimum, relative maximum, asymptote, multiplicity

Aspect of lesson setup	Details
Prior knowledge	Vocabulary to describe features of a graph What it means for an equation to be a "good fit" for the data Types of transformations

Lesson Procedures

Directions/Script
Launch (10 minutes)
Warm-up • The purpose of this warm-up is to allow students to consider a graph of data and the units in preparation for informally fitting functions to sets of data, which is a focus of the lesson. This warm-up prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved (MP1). • The specific data shown here is used in the following activity, where students decide which of two given functions best fits the data, and in a future lesson, where students transform a given equation so that the corresponding graph fits the data. • While students may notice and wonder many things about the graph, the shape of the data in the context is the important discussion point, making use of the structure of the graph and relating it to the graphs of functions they have seen in previous lessons (MP7). • Display the graph for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner, followed by a whole-class discussion.

Directions/Script

- Student Responses:
  - » Things students may notice:
    - › The data is decreasing.
    - › Something is cooling down from about 70° to about 45°.
    - › The temperature is decreasing as time goes on, but it's decreasing faster at the start.
  - » Things students may wonder:
    - › What is the value when h = 0?
    - › What sort of a function would model the data?
    - › Why is there an almost 3 hour gap in the data?
    - › Will the points ever intersect the horizontal axis?
- Activity Synthesis: Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the graph. After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list that you are wondering about?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.
- If the general shape of the data or a possible situation that the data represents does not come up during the conversation, ask students to discuss these ideas.

Explore (30 minutes)

Which Function?

- Building on the work in the warm-up, the purpose of this activity is for students to determine which of the given functions is a better fit for the data. For this activity, students should use any language that makes sense to them to describe why a function is or is not a good fit and how they would change the function to be a better fit for the data. Since both functions offer reasonably good fits for the data, students have the opportunity to make an argument about why they think a particular function is a better fit (MP3).
- Monitor for students using different explanations of what makes a function fit the data, such as by focusing on the general shape, the accuracy for individual points, or the average error for all the points.
- This activity works best when each student has access to devices that can run the Desmos applet because students will benefit from seeing the relationship in a dynamic way. If students don't have individual access, projecting the applet will be helpful during the synthesis.
- *Writing, Conversing: MLR1 Stronger and Clearer Each Time*. Use this routine to help students improve their writing by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with two to three partners to share their response to the question "Which function better fits the shape of the data? Explain your reasoning." Students should first check to see if they agree with each other about which function fits the data better. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, "What did you look at in the graph to make your decision?" or "Can you say more about what _____ means?" Next, provide students with 3–4 minutes to revise their initial draft based on feedback from their peers. This will help students produce justifications for determining what makes a function a good fit for data. Design Principle(s): Support sense-making; Optimize output *(for explanation)*

Directions/Script

- Student Response:
  1. Sample responses: *f* fits better because it stays closer to the data points. *g* fits better because its shape is a better fit and the vertical intercept is closer to what the data shows.
  2. Answers vary.
  3. Sample response: Adding the 45 raises the *T* value, the output, of each function by 45 degrees, so the graph is 45 degrees higher at each point.
  4. Sample response: *g* would fit better if the curve was moved up higher about 3 degrees.
- Anticipated Misconceptions: In order to show the temperature trend better, the first tick mark on the temperature axis represents 45 degrees, even though each successive tick mark only represents an additional 5 degrees. If students are confused that the first tick mark does not represent 5 degrees, remind them that since the range of this function does not include any numbers less than 45, it is convenient to start the range values at 45.
- Activity Synthesis: Using graphing technology, project the data given here, along with the two functions *f*(h) and *g*(h).
- Invite previously identified students to share which function they think fits better and why. Since there is no single correct answer, attend to students' explanations and ensure the reasons given are correct. Ask two to three students for ideas on how they would adjust either *f* or *g* to be a better fit.
- Conclude the discussion by showing how the graphs of *f* and *g* change when the 45 is removed from the equation. If students called the 45 the vertical intercept, note that this is true for some equations, such as the b in y = mx + b, but the constant term is not always the vertical intercept, as shown by the equations for *f* and *g*. Tell students that a goal of this unit is to understand how to transform the graphs of functions in different ways and what different transformations mean for the corresponding expressions.

Directions/Script

What happened to the graph?

- In this partner activity, students take turns describing transformations of a graph and sketching the transformed graph from the description. As students trade roles explaining their thinking and listening, they have opportunities to refine and use more precise language when describing transformations (MP6).
- Encourage students to attend to details such as direction, distance, and shape. While students work, record words you notice students using in their descriptions, such as "vertex," "intercept," or "maximum," for all to see to provide ideas for other students and to reference during the whole-class discussion.
- Arrange students in groups of two. Tell students that they are going to take turns. One partner will describe the transformation of Graph a to Graph b that they see on their handout, and the other will draw the transformation based on the description. Each partner will draw three graphs and describe three transformations.
- Ask students to be specific in their descriptions, but note that the goal is for their partner to draw the transformation correctly without needing to name specific points.
- Distribute two half sheets to each group from the blackline master, one to each student. Remind students to keep their sheet hidden from their partner.
- *Conversing: MLR2 Collect and Display*. Listen for and collect language students use to describe the transformations. Record informal student language alongside the mathematical terms (translate up or down, translate right or left, reflect, stretched, squashed) on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will provide students with a resource to draw language from during small-group and whole-group discussions. Design Principle(s): Maximize meta-awareness; Support sense-making
- Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce remaining cards if time allows. Supports accessibility for: Conceptual processing; Organization
- Student Response:
  - » Sample responses
    1. Translated up by 3 units
    2. Translated right by 1 unit
    3. Translated left by 1 unit and down by 2 units
    4. Reflected across the y-axis
    5. Squashed horizontally by a factor of 1/2
    6. Stretched horizontally by a factor of 2 or squashed vertically by a factor of 1/4

Directions/Script
• Anticipated Misconceptions: Some students may describe the transformations without enough detail, making it difficult for their partner to sketch the correct transformation. Emphasize that the goal is for their partner to draw the transformed graph with precision so that it matches what they see exactly. • Activity Synthesis: The purpose of this discussion is for students to describe the transformations they saw when graphing. Encourage students to use precise language such as translate, reflect, and stretch. Students will continue to refine their language around graphical transformations throughout the unit, so it is okay for students to use more informal language at this time. • Begin the discussion by inviting students to share what types of transformations they saw, displaying the graphs for all to see to help illustrate student descriptions and connecting back to the list of words recorded during the activity. Connect any words students used back to geometry vocabulary (translate and reflect). Ask, "Are any of these transformations dilations?" (No, they are only stretching in one direction.)
Debrief and Closure (10 minutes)
Synthesis: Display the three graphs. Invite students to describe how to transform each graph to fit the points. Here are some questions for discussion: • "Which graph fits the data the best?" (All three can fit perfectly, for example, by translating the parabola up 2 and right 1.) • "Imagine a situation where each graph—linear, quadratic, and exponential—would be the best choice to model the relationship." (Any explanation works so long as the rate of change in the scenario matches the graph chosen. For example: The exponential would be best if the situation was population growth over time. The line would be best if the situation was price per pound.) • "When we fit a curve to data points, will the curve always go precisely through all of the points?" (Not always, but the shape of the curve should fit the general shape of the data.)
Cooldown: Sample responses • Translate the graph right 1 unit and up 1 unit. • Translate the graph right 2 units. • Translate the graph right 3 units and down 2 units. • Stretch the graph vertically by a scale factor of about 4.

Lesson C14. Tenet 2: Algebra 1—Creating Equations

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 2, which is described in Table 1.

Lesson C14: Table 1. Overview of Tenet

Essential tenet	# 2: Agency and Belonging
Student outcomes	5. Students feel safe to engage in productive struggle, evidenced by their willingness to take risks, make errors, receive feedback, and revise their thinking. This includes students sharing their unfinished thinking and celebrating their progress and peers' work.
Guidelines	Support collaborative, peer-to-peer conversations that invite curiosity and inquiry, build collective understanding, and offer a judgment-free space where students feel comfortable offering ideas and answers they aren't sure about. (relationship skills, self-awareness, responsible decision-making)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C14: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	CCSS
Domain	A-CED
Content standards	**CCSS.Math.Content.HSA.CED.A.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **CCSS.Math.Content.HSA.CED.A.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
Standards for Mathematical Practice (SMP)	SMP 1. Make sense of problems and persevere in solving them. SMP 3. Construct viable arguments and critique the reasoning of others. SMP 6. Attend to precision.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C14: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	• **Social Awareness:** The abilities to understand the perspectives of and empathize with others, including those from diverse backgrounds, cultures, and contexts. • **Relationship Skills:** The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
Example(s)	• Students are asked to value the solutions reached by their peers and the moral justifications that support them. • Students work in pairs and small groups and then engage with the whole class. Students need to effectively communicate with peers who are diverse in identity, learning style, and so on.

Lesson Overview and Goals

We explore how to use the distance formula to determine where the best (as defined by students) location is to build a new grocery store, considering the location of three neighborhoods serving groups of varying socioeconomic status, and consider the concept of fair through the distinction of what is equal versus what is equitable.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C14: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	By the end of the lesson, students will know or be able to use the distance formula and apply it to justify their answer to a question about fairness.
Key vocabulary	Distance, Pythagorean theorem
Prior knowledge	Apply the Pythagorean theorem to find the distance between two points on a graph.

Lesson Procedures

Directions/Script
Launch (10 minutes)
In this lesson, students will explore a problem without one correct solution. Their solution must be mathematically correct, but beyond that, they will need to defend their solution based on what they believe is either the most equal or most equitable approach. Have students use a think-pair-share to try defining the terms "equal" and "equitable" and then try identifying a situation that would be equal but not equitable. If students struggle trying to define the term "equitable," you can let them know that it might be substituted with the word "fair." An example: • Two siblings are 3 years apart in age. If they have the same bedtime, that is equal, but it's equitable for the older sibling to have a later bedtime.
Have students pair with someone else to discuss this prompt and then discuss as a class. Prompts to ask might include the following: • What would be an equitable approach to splitting up the pizza? • What are some different ways to share the pizza that might be considered equitable? • Would it be equitable to only share the pizza with the two hungriest people, or perhaps give the entire pizza to the person who hasn't eaten since yesterday?
Ask the class how to find the missing side of a right triangle. Students should identify the need to use the Pythagorean theorem. Teacher should model how to solve the white triangle and find the length of Side A. Students should work alone or in pairs/groups to find the other missing sides while the teacher circulates around the classroom to help by offering prompts: • How would you set up the Pythagorean theorem for this triangle? Which information goes where in the equation? • If you're trying to find Side D, what information would go where into the Pythagorean theorem? (Hint: it's not possible to find D because there isn't enough information). As students try to identify which side is the longest, they may be tempted to look at which is visually the longest (in this case, Side D). That's a reasonable answer to the question, provided that students understand they can't truly compare the length of D to A, B, or C because they don't know the numerical distance of Side D. Listen for students who might try to justify that A, B, or C is longest because of its visual distance, although by applying the Pythagorean theorem, they should be able to determine which is mathematically longest.

Directions/Script
Explore (30 minutes)
Start by sharing the slide with the grid. Students should understand that this grid is considered Center City and that the Red Star Supermarket is at the intersection of the two lines. Ask students how to find the distance from A to the Red Star Supermarket. Students are likely to identify the need to use the Pythagorean theorem but may be confused because there isn't an axis. Teacher might ask, "Would it help to have an axis? Where would you draw it?" (Note that students could draw axes in different places, and the distances will be the same.) Remind or help students recognize that they can use the distance formula (derived from the Pythagorean theorem) to find distances between two known points. Choose a student to model the process for finding the distance from A. Then have students work alone or in pairs/groups to calculate the distance from each B and C to the supermarket. Choose a student(s) to read the prompt on the next slide while others read it to themselves. Students might be asked to explain to a partner the prompt in their own words. Teacher can also ask a student(s) to put the prompt into their own words so the teacher can check for understanding. Students might be confused by the word "best." Teacher can explain that they get to define what best means in this context and that it could mean what is equal or what is most equitable. Some considerations: • The different neighborhoods (A, B, and C) have different poverty rates. Should the relative poverty rates for each of the neighborhoods be taken into consideration? Or just for two of the neighborhoods? Would it be fair to only consider the needs of one neighborhood? • How should students take into consideration the location of the Red Star Supermarket in relation to the three neighborhoods? What about in relation to the location of the new market (e.g., would it make sense to place the second supermarket very close to the first, even if that location would seem most fair given the needs in the different neighborhoods)?
Debrief and Closure (10 minutes)
This slide presents an opportunity to explore the concept of "food deserts," which is what the previous activity began to touch on. Teacher should facilitate small-group/whole-group discussions about the questions on the slide. Although the image doesn't necessarily demonstrate unequal/inequitable access to healthy food by race or socioeconomic status, it demonstrates how rates/maps (distance) can be used to understand inequitable access across different neighborhoods. Some issues that the teacher might want to add to the discussion include the following: • "I wonder what the racial composition is in each of these different neighborhoods?" • "I wonder how this map compares to other major metropolitan areas?" • "I wonder what people in the outer ring neighborhoods do for food instead, and how does their limited options impact other things like health?"

Directions/Script
The data in this chart demonstrate that across all three socioeconomic status categories, the average number of grocery stores is always lowest in Black communities. Teacher can engage students in a discussion about the prompt or ask each student to write a response that they then share. Another prompt could be: "How does the information relate to the other activities we've done in class?"

Lesson C15. Tenet 3: Algebra 2—Exponential Functions

This lesson was created by a teacher during a design lab focused on culturally responsive and sustaining education (CRSE) and social and emotional learning (SEL). The lesson aligns with Tenet 3, which is described in Table 1.

Lesson C15: Table 1. Overview of Tenet

Essential tenet	#3: Collective Responsibility
Student outcomes	1. Students possess and use interpersonal skills to engage in mathematical discourse and communicate their ideas and findings.
Guideline	(a.) Develop regular routines and a scaffolded structure for students to provide thoughtful, descriptive feedback and support to one another about their mathematical thinking and work. (self-awareness, self-management, social awareness, relationship skills)

Table 2 lists the mathematical standards to which this lesson aligns.

Lesson C15: Table 2. Mathematical Standards Related to This Lesson

Component	Details
Source	Common Core State Standards
Domain	Functions
Content standards	**HSF-LE.A.1.b:** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **HSF-LE.A.1.c:** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **HSF-LE.A.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Standards for Mathematical Practice (SMP)	SMP 2. Make sense of problems and persevere in solving them. SMP 5. Use appropriate tools strategically. SMP 8. Look for and express regularity in repeated reasoning.

Table 3 lists the SEL competencies to which this lesson aligns.

Lesson C15: Table 3. SEL Competencies

Component	Details
Source	CASEL
Core competency and descriptor(s)	**Relationship Skills:** The abilities to establish and maintain healthy and supportive relationships and to effectively naavigate settings with diverse individuals and groups.
Example(s)	• Communicating effectively • Practicing teamwork and collaborative problem-solving • Showing leadership in groups • Seeking or offering support and help when needed

Lesson Overview and Goals

Learning Goals: Compare and contrast (orally) exponential growth and decay. Determine values of simple exponential functions in context.

Lesson Setup

Table 4 provides some key information to help teachers set up and carry out the lesson.

Lesson C15: Table 4. Lesson Setup and Logistics

Aspect of lesson setup	Details
Lesson source/resource	Slides
Student outcomes	Students will be able to calculate values that are changing exponentially.
Key vocabulary	Exponential growth, exponential decay, growth factor
Prior knowledge	• Exponential change involves repeatedly multiplying a quantity by the same factor rather than adding the same amount. • Exponential growth happens when the factor is greater than 1, and exponential decay happens when the factor is less than 1. • A quantity that grows exponentially may appear to increase slowly at first but then increases very rapidly later.

Lesson Procedures

Directions/Script
Launch (10 minutes)
• This warm-up recalls previous work on arithmetic and geometric sequences. While work in this unit will focus on quantities that change exponentially, it is also important for students to understand how these differ from quantities that change linearly. Have two students read the definitions of arithmetic and geometric sequences to activate prior knowledge. • Give students 5 minutes to independently create examples of bank accounts that demonstrate an arithmetic and geometric sequence. • Invite students to share their examples with a partner first, and then with the whole class, highlighting that successive numbers in an arithmetic sequence have equal differences, while successive numbers in a geometric sequence have equal quotients.
Explore (30 minutes)
Shrinking a Passport Photo • The goal of this task is for students to analyze exponential growth in the context of successive scaling. • Students may use a range of strategies, but it is fine if the only approach students come up with is listing out successive values. Monitor for students who use one or more of the following strategies to share during the discussion: » Calculate each successive height of the scaled passport photo and list them as a sequence (of decreasing heights), arrange them in a table, or show each measurement in the diagram. (Some students may choose to use a spreadsheet to perform the calculations.) » Write an expression that shows repeated multiplication. » Think about powers of 4/5 or of 0.8 and write an expression. » Write a function whose input is the number of scalings and whose output is the corresponding height of the passport photo. • Tell students that they will view a short video. Give students 1 minute of quiet time to think and ask them to be prepared to share at least one thing they notice and one thing they wonder. • Invite students to share the things they noticed and wondered. Tell them that they will investigate how many times an image needs to be reduced in size to fit passport requirements. Provide access to rulers if requested. • *Representation: Internalize Comprehension*. To support working memory, provide students with graph paper. Encourage students to illustrate their strategies. Demonstrate drawing successively reduced rectangles. Then, invite them to annotate their work by showing the calculations used to arrive at each successive reduction. For example, drawing arrows between the rectangles that show the dimensions being multiplied by 0.8. If students indicate the desire to derive an expression from their patterns, encourage them to also record their thinking as they develop this strategy. *Supports accessibility for: Memory; Organization*

Directions/Script

- Student Responses:
  1. a. About 77 mm. Sample reasoning: $150 \cdot (0.8)^3 = 76.8$.
     b. Approximately 40 mm. Sample reasoning: $150 \cdot (0.8)^6 \approx 39.3$.
  2. 7 times. Sample reasoning: $150 \cdot (0.8)^7 \approx 31.5$.
  3. 9 times. Sample reasoning: $150 \cdot (0.8)^8 \approx 25.2$ and $150 \cdot (0.8)^9 \approx 20.1$.
- Anticipated Misconceptions: If students struggle interpreting what it means to scale the photo by 80%, ask them to consider what would happen to the dimensions if the photo were doubled in size? Halved? Then ask them what percent scaling each of these corresponds to (200% and 50%).
- Activity Synthesis: Invite previously identified students to share their responses in the order shown above. The order shows an increasing level of abstraction in representing the height of the passport photo after repeated scaling.
- Help students connect the different strategies by highlighting the repeated multiplication by the same factor in each strategy. If not mentioned by students, reinforce the fact that repeated multiplication can be expressed efficiently with an exponent.
- If time permits, consider showing a graph to illustrate how the height of the photo changes as a result of successive scaling.
- *Speaking: MLR8 Discussion Supports.* As students share their strategies for scaling the image by 80% multiple times, press for details by asking how they know that the repeated scaling of the image can be represented by an exponential expression. Show concepts multimodally by writing an expression that shows repeated multiplication and an equivalent expression with an exponent. This will help students justify why the situation can be represented by an exponential expression. *Design Principle(s): Support sense-making*

Pond in a Park

- This activity prompts students to examine a different exponential change situation involving algae growth. Students are not specifically asked to produce expressions since the focus of this activity is on the different ways to reason about an exponential function, but some may choose to do so.
- Here, time is the independent variable, and students think about it in two directions: recognizing that when time goes forward, the area covered by the algae doubles each day, while going back in time, the area covered by the algae is multiplied by 1/2. During the whole-class discussion, students consider how to calculate how much of the pond is covered over the course of half a day. Considering this type of calculation now in an informal way helps prepare students for future lessons in which they will learn how to calculate this type of value exactly.
- Monitor for students using tables or other organizing strategies to determine the amount of the pond covered on different dates.
- Making graphing or spreadsheet technology available gives students an opportunity to choose appropriate tools strategically (MP5).
- Ask students to read the paragraph in the Task Statement. To help them make sense of the context, consider introducing a simple diagram of the pond and asking students to try showing how the algae might be growing. There are many ways to do this, and figuring out how to draw it is an interesting challenge that requires making some assumptions and decisions. For example, students could start with a drawing that shows the pond on May 18 and move forward in time, or start with a drawing that shows the pond on May 24 (when it is entirely covered by algae) and move backward in time.
- Once students show an awareness of what is happening in the scenario, ask them to proceed with the activity.

Directions/Script

- *Representation: Develop Language and Symbols*. Display or provide a timeline or calendar with the days from May 12 through May 24. Demonstrate drawing a circle to represent the pond and shading in the circle accordingly to illustrate the algae on a given day. Begin with the given information, placing an entirely shaded circle on May 24. Allow students to use this method independently or scaffold the activity further by pausing after each question and eliciting student input to demonstrate adding a new shaded pond symbol for each question. *Supports accessibility for: Conceptual processing; Memory*
- Student Responses:
  1. May 23
  2. Sample responses: By May 18 only 1/64, or about 1.5% of the area of the pond is covered.
  3. $1/2^{12}$ of the area of the pond, or about 0.02%. Sample response: One day before May 24, 1/2 of the pond is covered. Two days before May 24, $1/2^2$ of the pond is covered. Three days before May 24 $1/2^3$ of the pond is covered, and so on. May 12 is 12 days before May 24.
- Anticipated Misconception: Some students may think the growth of algae is linear and conclude that the pond is half covered on May 18, halfway between May 12 and May 24. Ask them to consider what happens on May 19 in that case, or ask them to draw how the area covered changes from May 12 to May 18.
- Activity Synthesis: The goal of this discussion is to get students thinking about how to determine values of exponential functions when the input is not an integer. Invite previously identified students to share their responses to the questions, displaying any organization strategies used for all to see.
  - » Ask, "If 50% of the pond is covered at the start of May 23, how would you figure out how much of the pond is covered halfway through May 23?" After a brief quiet think time, select students to share their strategies. Some possible suggestions:
    - › Graph the percent coverage over time for the pond and use it to estimate.
    - › Make a table for days since May 12 and percent of the pond covered and use it to estimate.
    - › Write an exponential equation for the situation where the input is days since May 12 and the output is the percent covered and evaluate the equation for 11.5 days.

There is no need to discuss any of these strategies not brought up by students at this time.

- *Writing, Listening, Conversing: MLR1 Stronger and Clearer Each Time*. Use this routine to provide students with multiple opportunities to clarify their explanations through conversation. Give students time to meet with two to three partners to share their response to the last question. Give students time to meet with two to three partners to share and receive feedback on their responses. Display feedback prompts that will help students strengthen their ideas and clarify their language. For example, "Your explanation tells me ...," "Can you say more about why you ...?" and "A detail (or word) you could add is _____ because. ..." Invite students to go back and revise or refine their written responses based on the feedback they received. *Design Principle(s): Optimize output (for justification); Cultivate conversation*

Directions/Script
Debrief and Closure (10 minutes)
Synthesis: Ask students, "How are the shrinking passport photo and growing algae situations similar?" Highlight: • In each case, there is a sequence of numbers that changes in a predictable way. • The numbers make a geometric sequence: Each number is always multiplied by the same factor to get the next one. • These situations can be represented by exponential expressions or equations. • The relationship between the two quantities in each situation can be modeled by an exponential function. Ask students, "How are the shrinking passport photo and growing algae situations different?" Highlight: • The height of the passport photo gets smaller while the area covered by the algae grows. • The height of the passport photo changes by a factor that is less than 1. The area covered by the algae grows by a factor that is greater than 1. • The pond is eventually full of algae while the passport photo can continue to shrink. • The height of the passport photo changes in a discrete way (forming a geometric sequence). The area of the algae changes continuously (that is, the area does not suddenly double at a particular point in time). Explain that we will explore many more situations involving exponential change in this unit and use exponential functions to solve problems.
Cooldown: • If needed, explain or demonstrate with a physical example that the visible area of the paper is the area of the resulting rectangle if the paper stays folded. • Student Response: 1. Disagree. After one round of folding, the visible area of the folded paper is 150 square inches. After two rounds, it will be a third of 150, or 50 square inches. 2. 50/9 or 50 • 1/3 • 1/3 square inches (or equivalent)